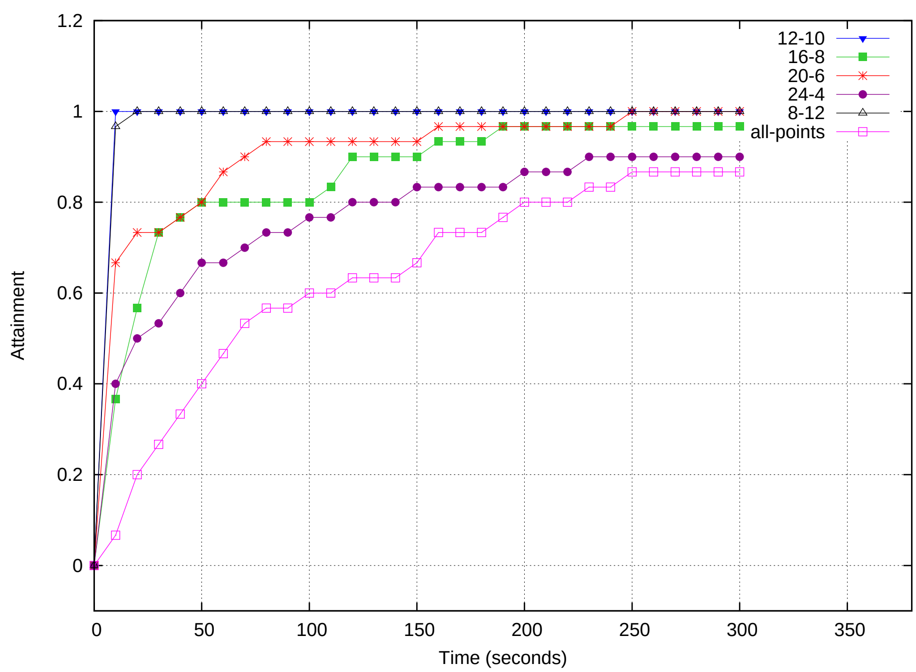
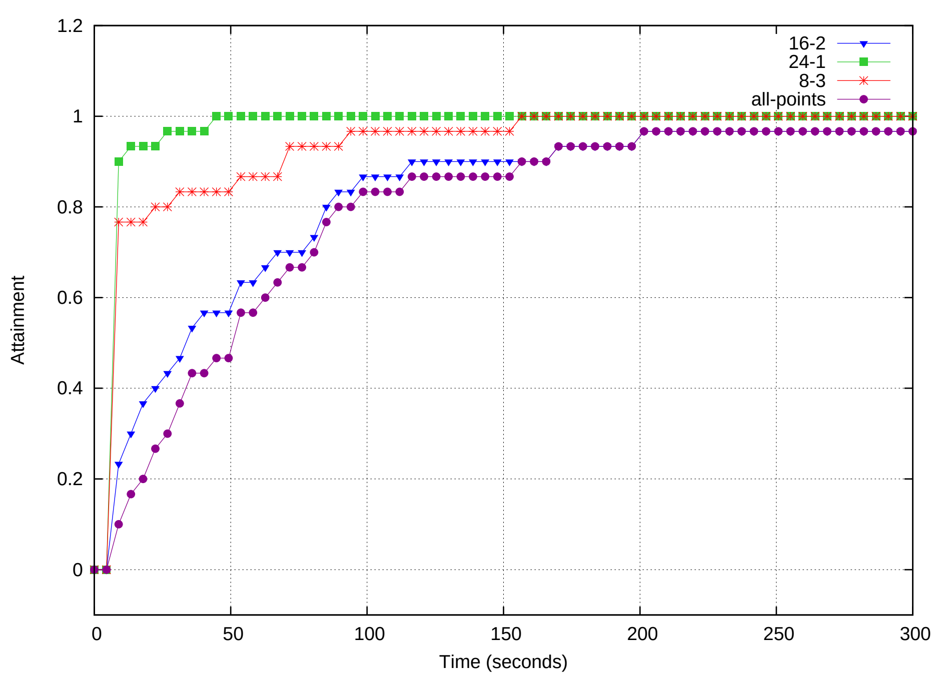

1.2
1
0.8
0.6
0.4
0.2
0
Attainment
0
50
100
150
200
250
300
350
Time (seconds)
12-10
16-8
20-6
24-4
8-12
all-points



16-2
24-1
8-3
all-points
Attainment
Time (seconds)
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150
200
250
300
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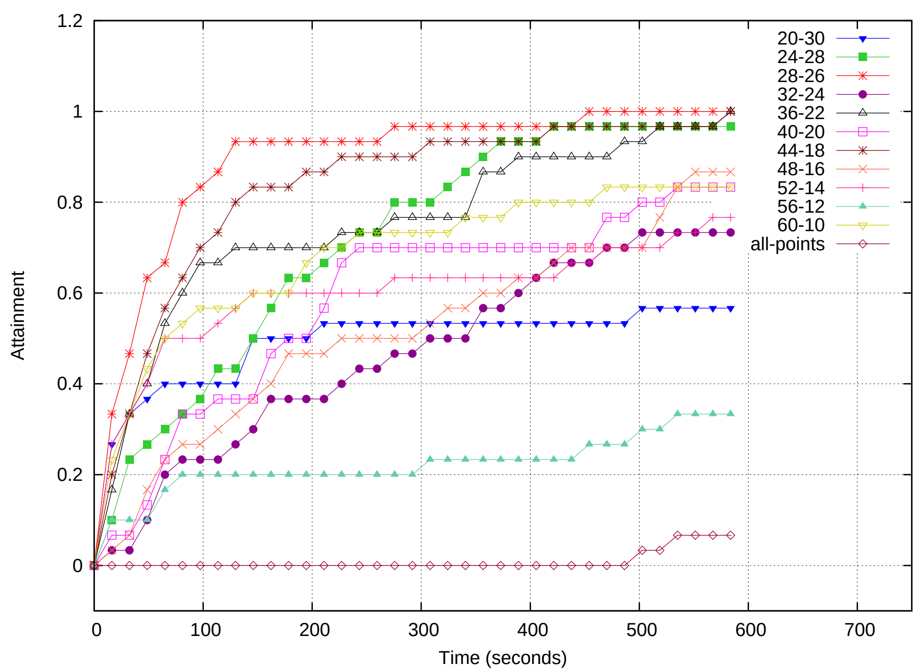
1
0.8
0.6
0.4
0.2
0



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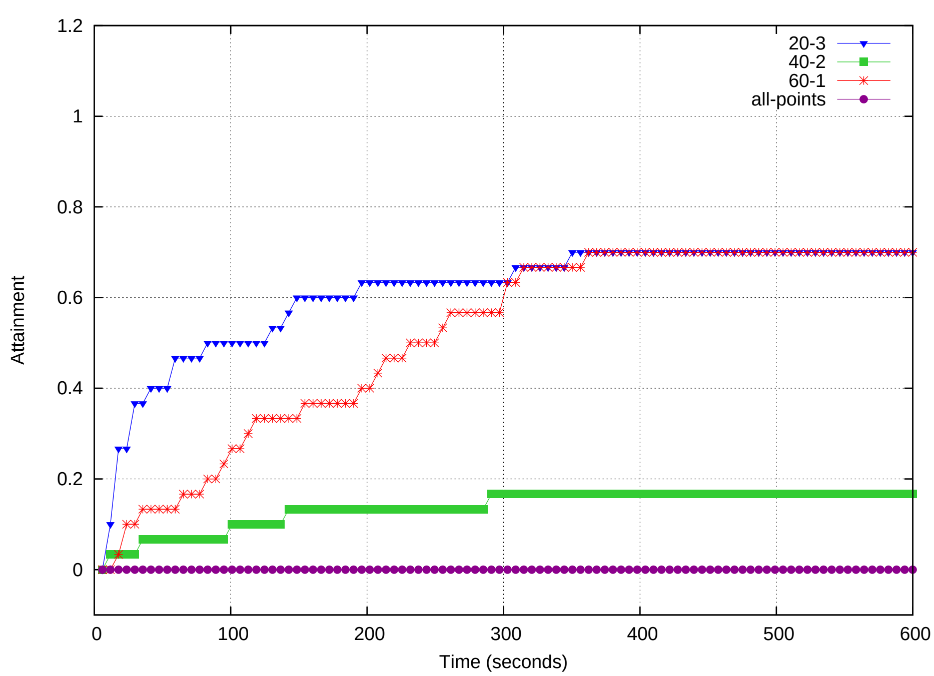
0.8
0.6
0.4
0.2
0
0
100
200
300
400
500
600
700
Attainment
Time (seconds)
20-30
24-28
28-26
32-24
36-22
40-20
44-18
48-16
52-14
56-12
60-10
all-points



20-3
40-2
60-1
all-points
Attainment
Time (seconds)
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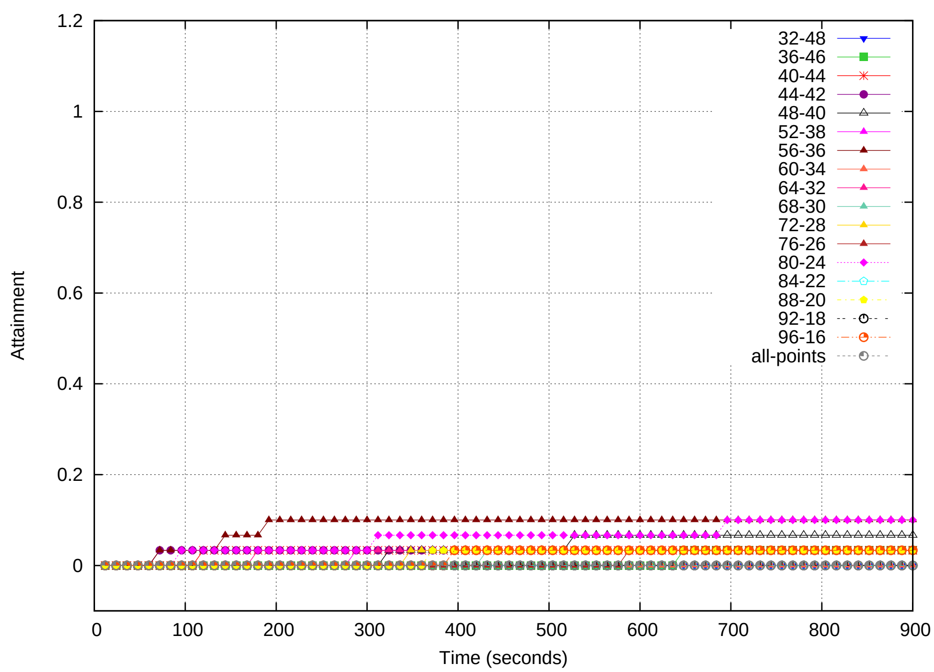
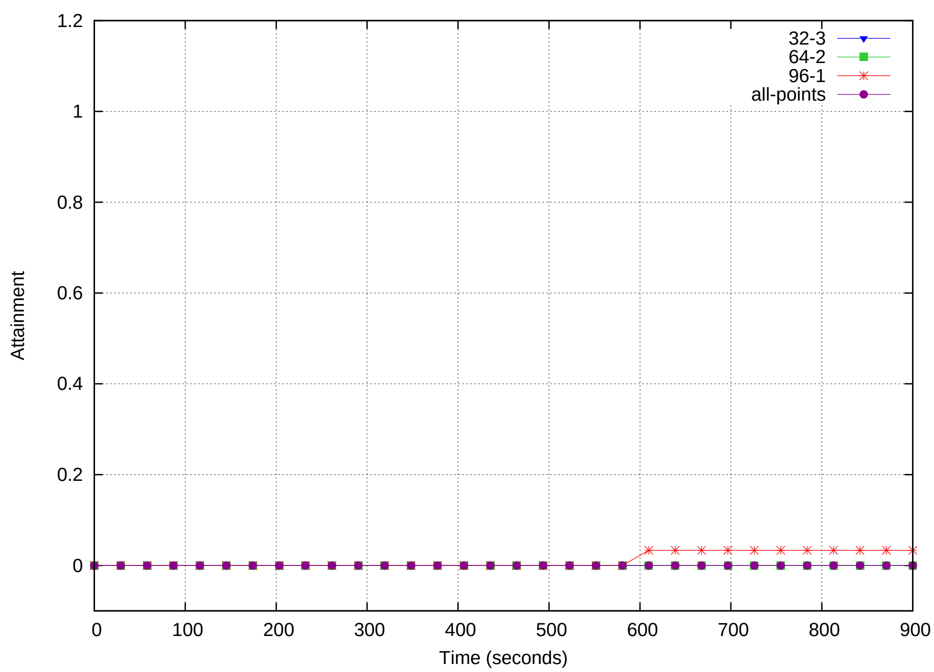
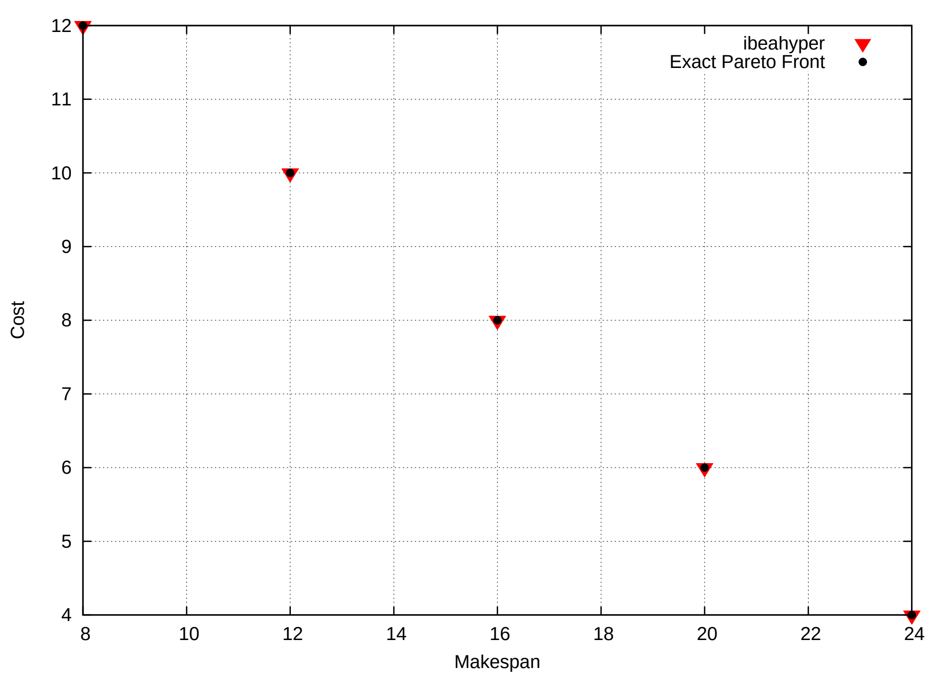
100
200
300
400
500
600
1.2
1
0.8
0.6
0.4
0.2
0



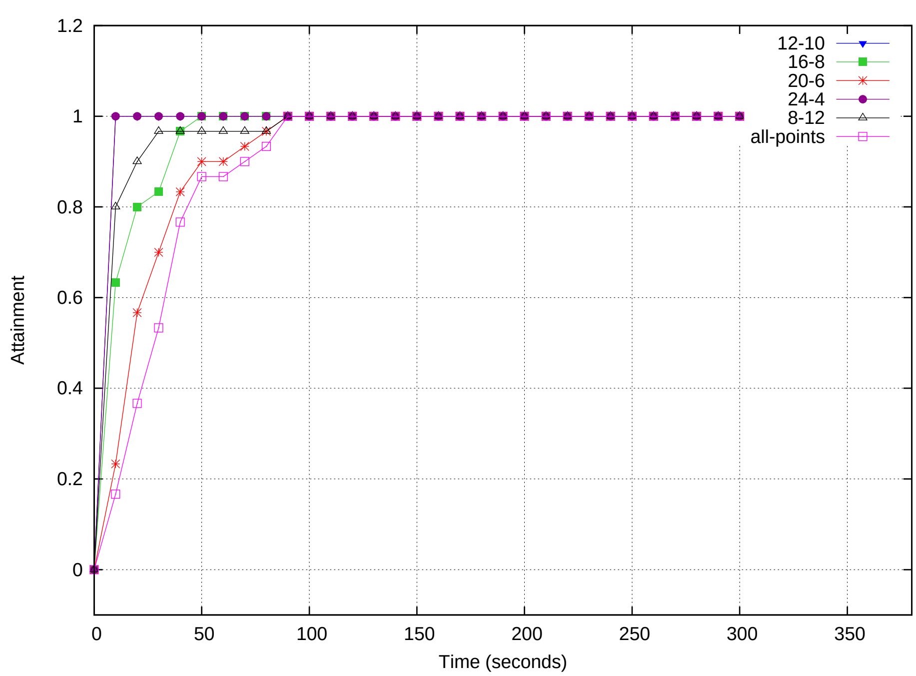
1.2
1
0.8
0.6
0.4
0.2
0
Attainment
0
100
200
300
400
500
600
700
800
900
Time (seconds)
32-48
36-46
40-44
44-42
48-40
52-38
56-36
60-34
64-32
68-30
72-28
76-26
80-24
84-22
88-20
92-18
96-16
all-points



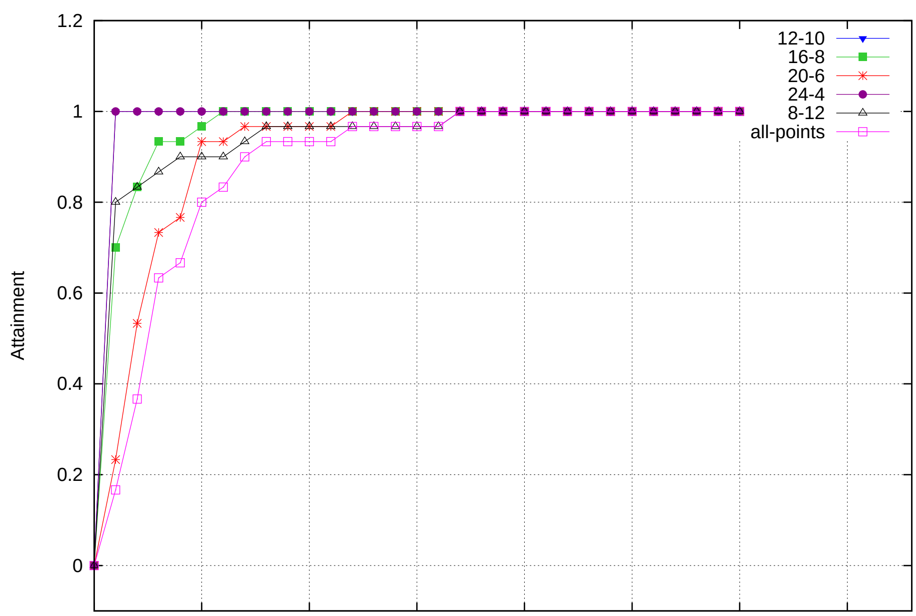
1.2
1
0.8
0.6
0.4
0.2
0
Attainment
0
100
200
300
400
500
600
700
800
900
Time (seconds)
32-3
64-2
96-1
all-points



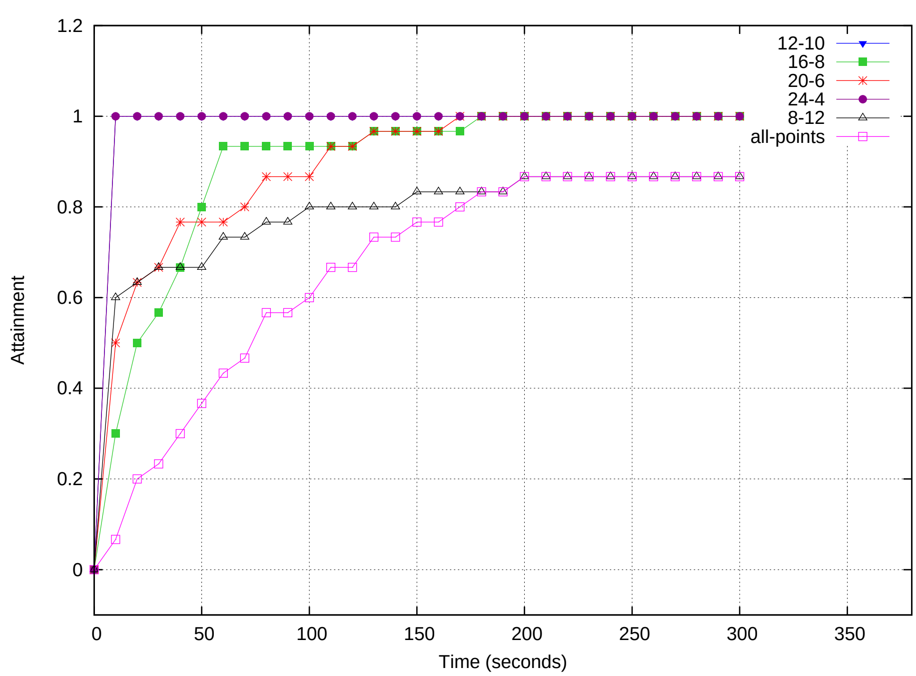
ibeahyper
Exact Pareto Front
Cost
Makespan
12
11
10
9
8
7
6
5
4
8
10
12
14
16
18
20
22
24

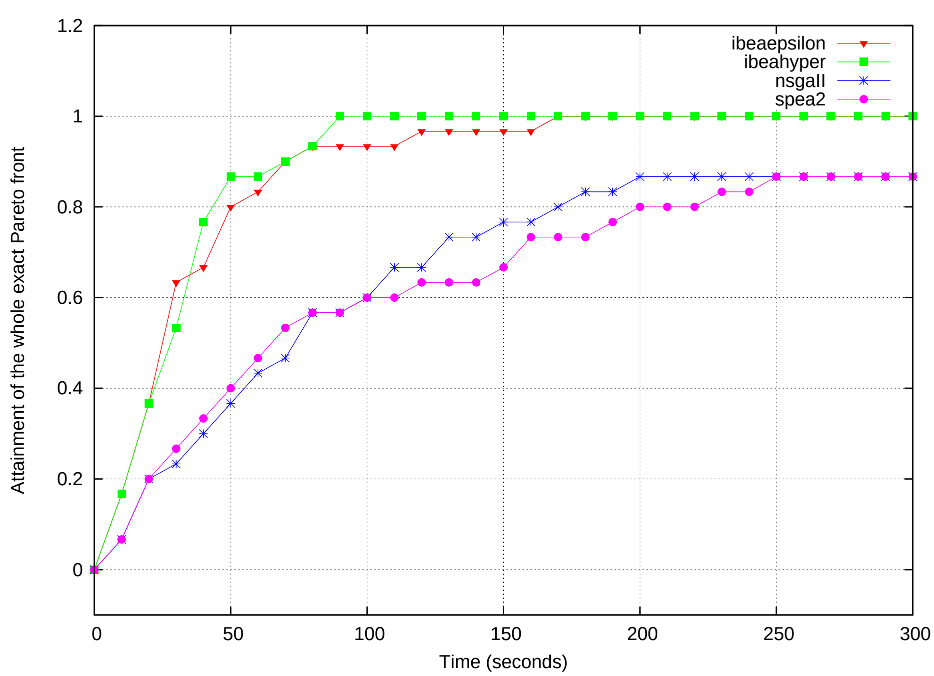


12-10
16-8
20-6
24-4
8-12
all-points
Attainment
Time (seconds)

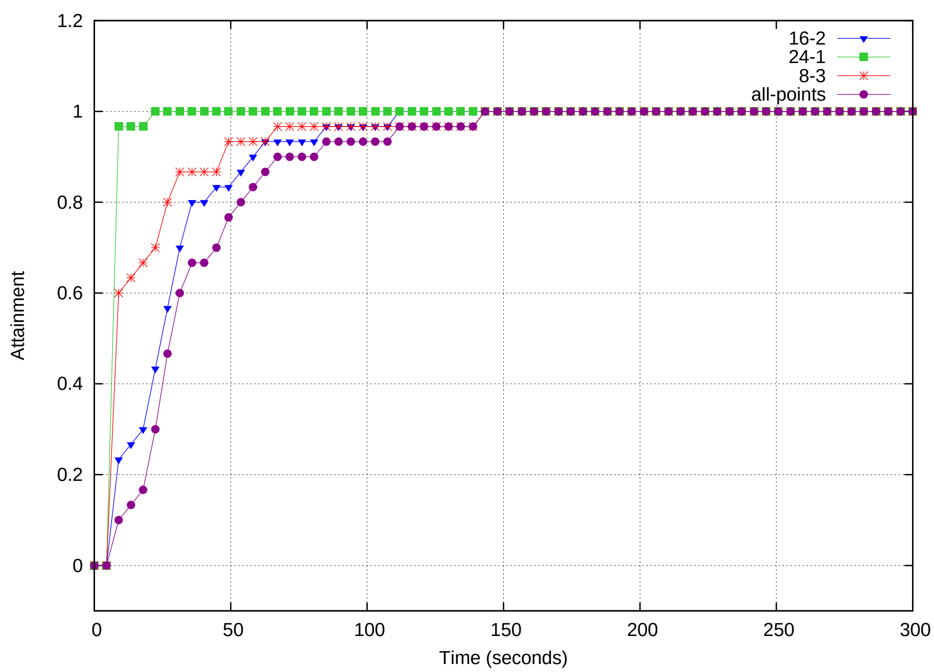


12-10
16-8
20-6
24-4
8-12
all-points
Attainment
Time (seconds)


1.2
1
0.8
0.6
0.4
0.2
0
0
50
100
150
200
250
300
350
Time (seconds)
Attainment
12-10
16-8
20-6
24-4
8-12
all-points



Attainment of the whole exact Pareto front
Time (seconds)
ibeaepsilon
ibeahyper
nsgaII
spea2
0
0.2
0.4
0.6
0.8
1
1.2
0
50
100
150
200
250
300


1.2
1
0.8
0.6
0.4
0.2
0
Attainment
0
50
100
150
200
250
300
Time (seconds)
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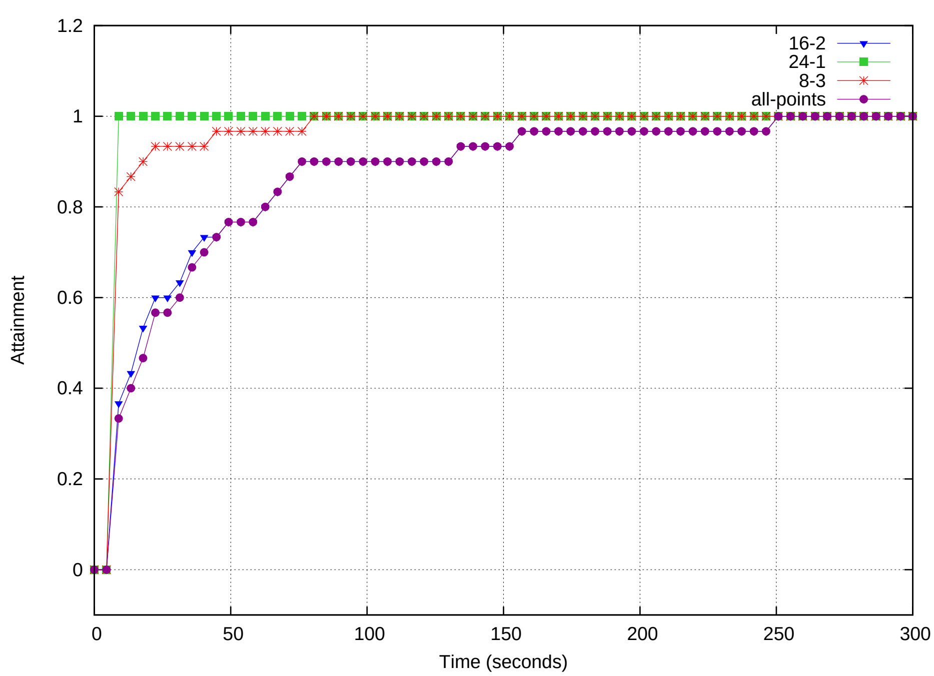
24-1
8-3
all-points



16-2
24-1
8-3
all-points
Attainment
Time (seconds)
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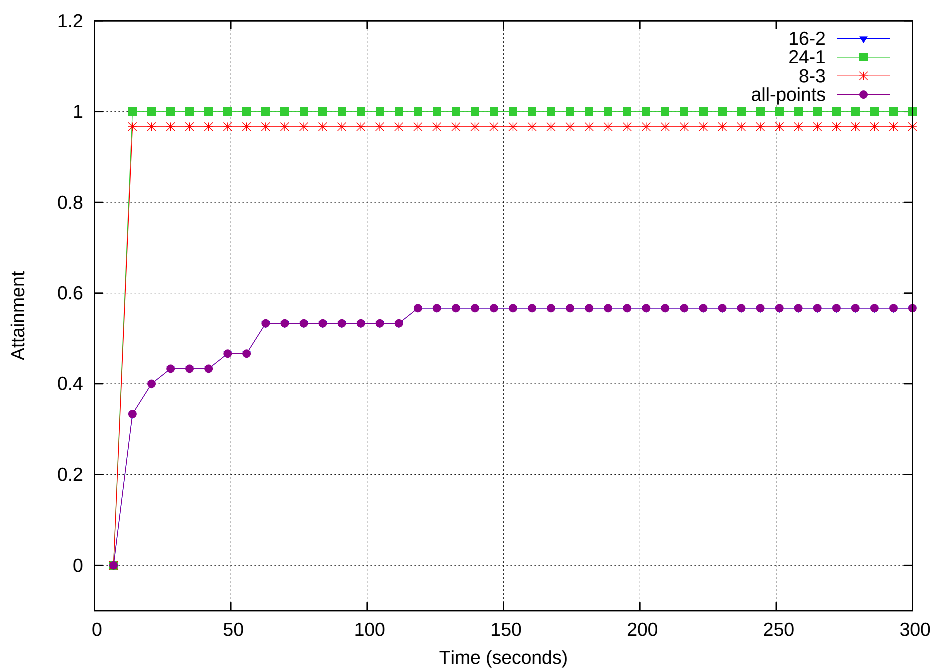
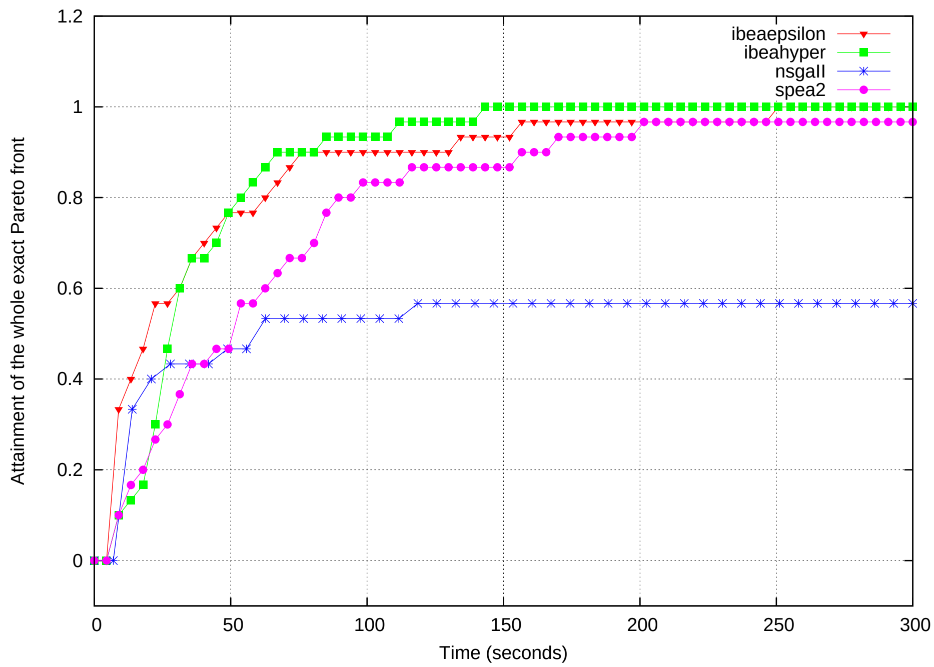
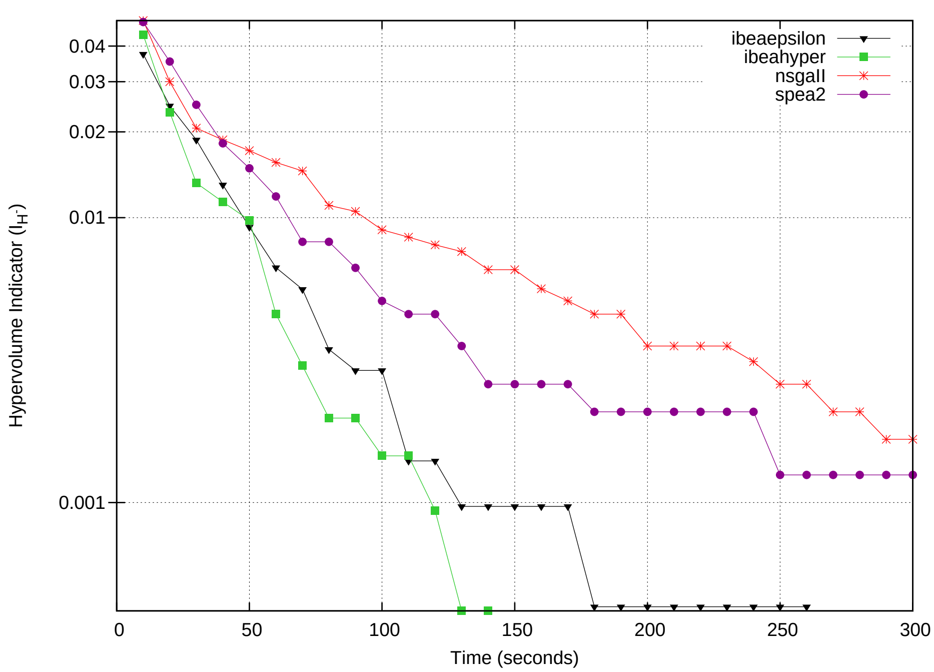
0.2
0.4
0.6
0.8
1
1.2
0
50
100
150
200
250
300



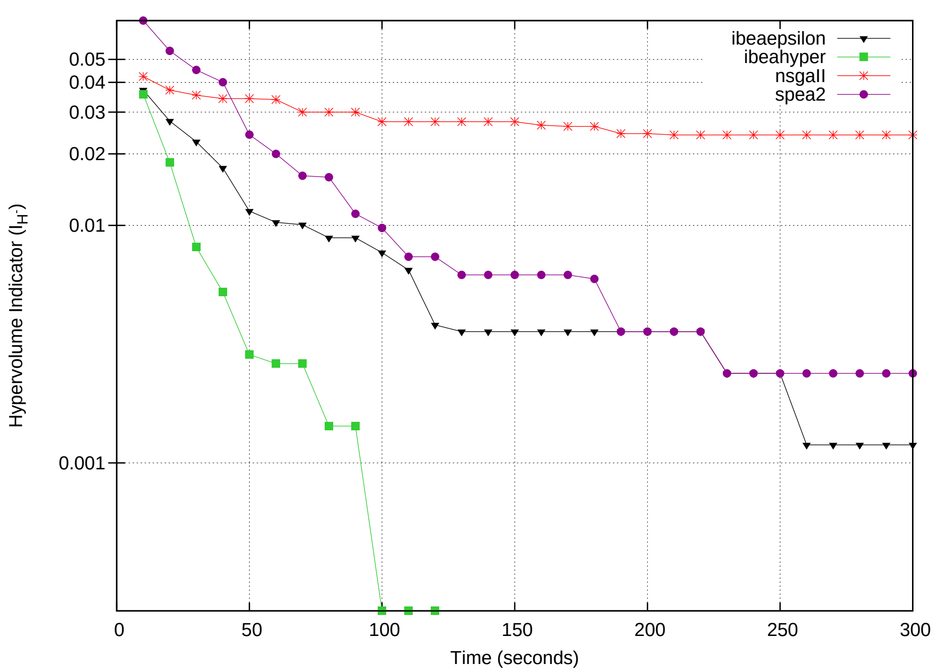
1.2
1
0.8
0.6
0.4
0.2
0
Attainment
0
50
100
150
200
250
300
Time (seconds)
16-2
24-1
8-3
all-points



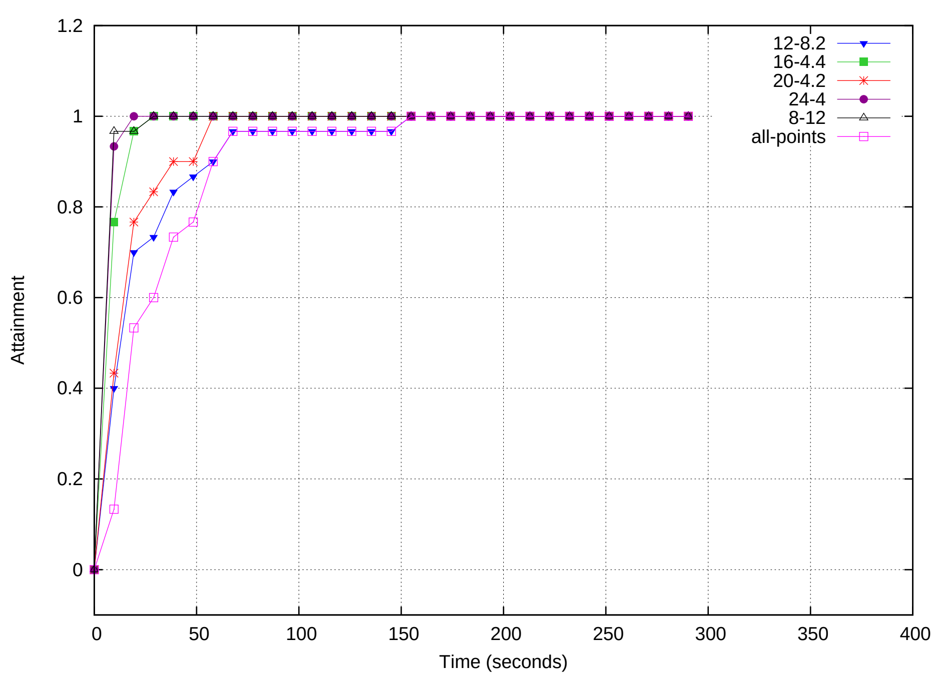
ibeaepsilon
ibeahyper
nsgaII
spea2
Attainment of the whole exact Pareto front
Time (seconds)
0
0.2
0.4
0.6
0.8
1
1.2
0
50
100
150
200
250
300



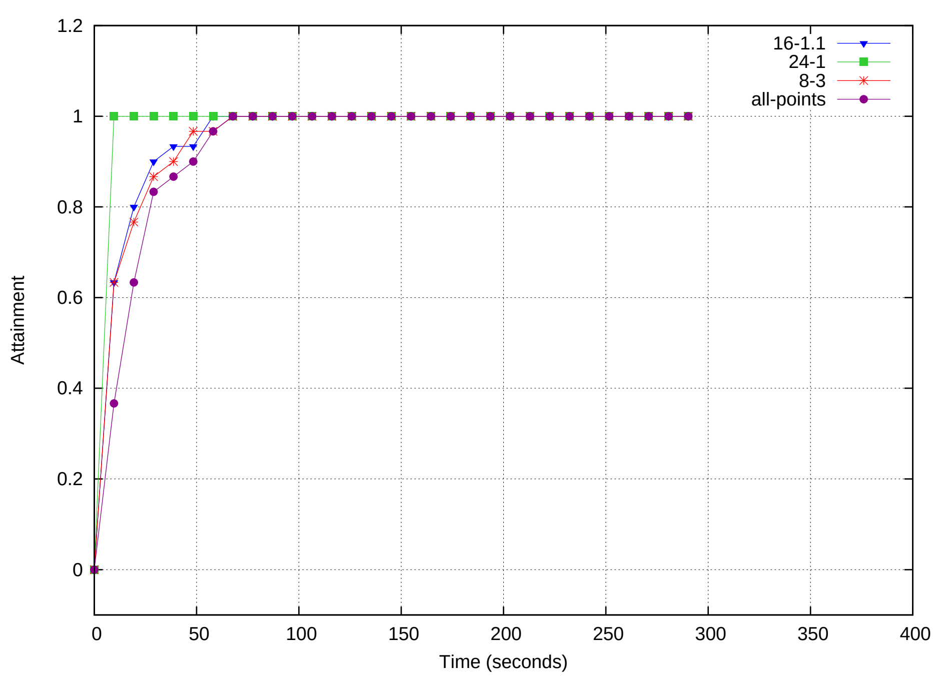
Hypervolume Indicator (I_{H^-})
0.04
0.03
0.02
0.01
0.001
0
50
100
150
200
250
300
Time (seconds)
ibeaepsilon
ibeahyper
nsgaII
spea2



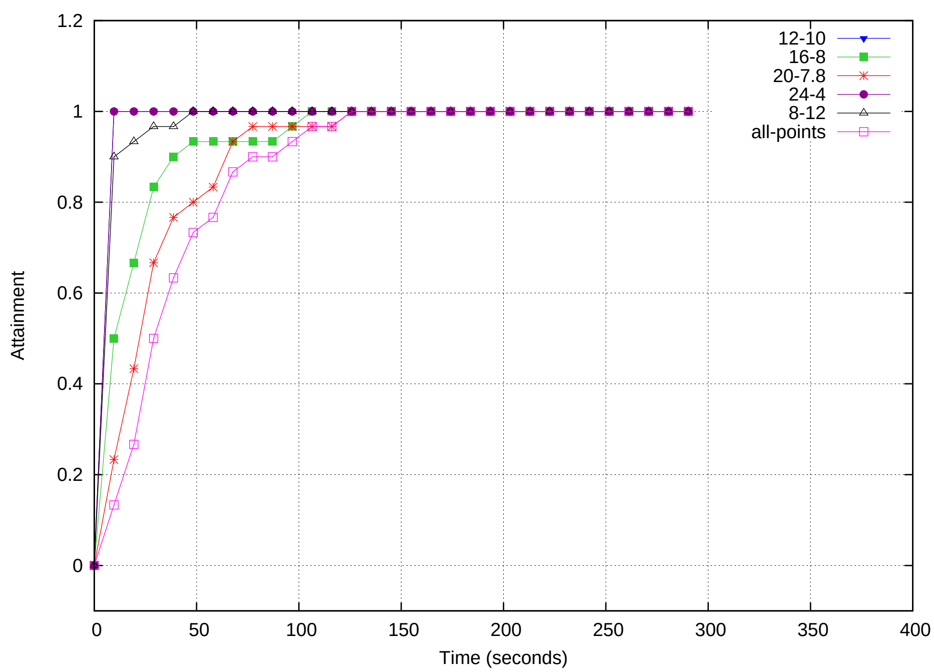
ibeaepsilon
ibeahyper
nsgaII
spea2
Hypervolume Indicator (I_{H^-})
Time (seconds)
0.05
0.04
0.03
0.02
0.01
0.001
0
50
100
150
200
250
300



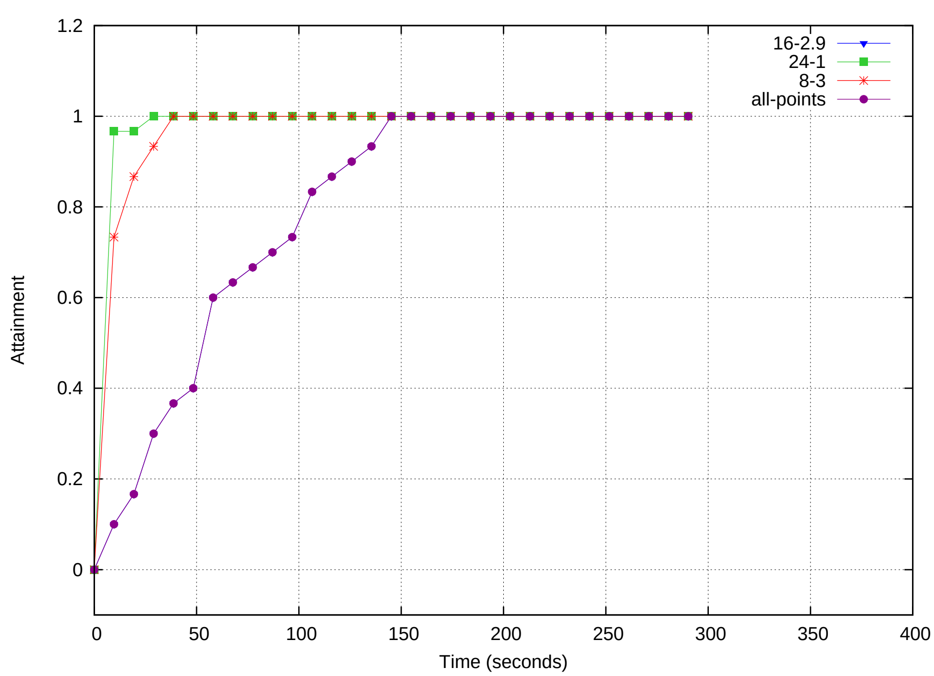
12-8.2
16-4.4
20-4.2
24-4
8-12
all-points
Attainment
Time (seconds)
0
0.2
0.4
0.6
0.8
1
1.2
0
50
100
150
200
250
300
350
400

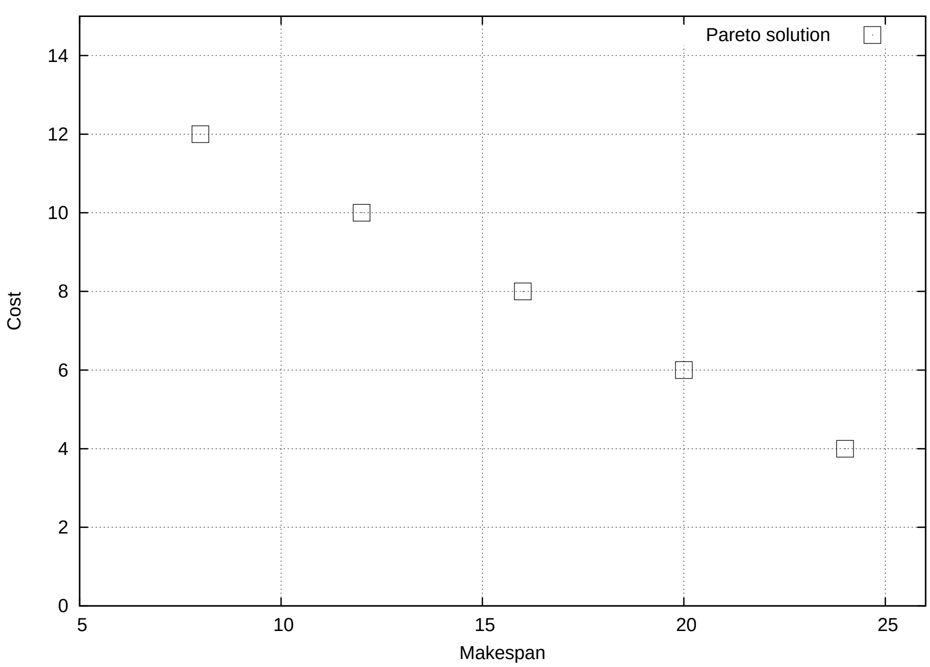


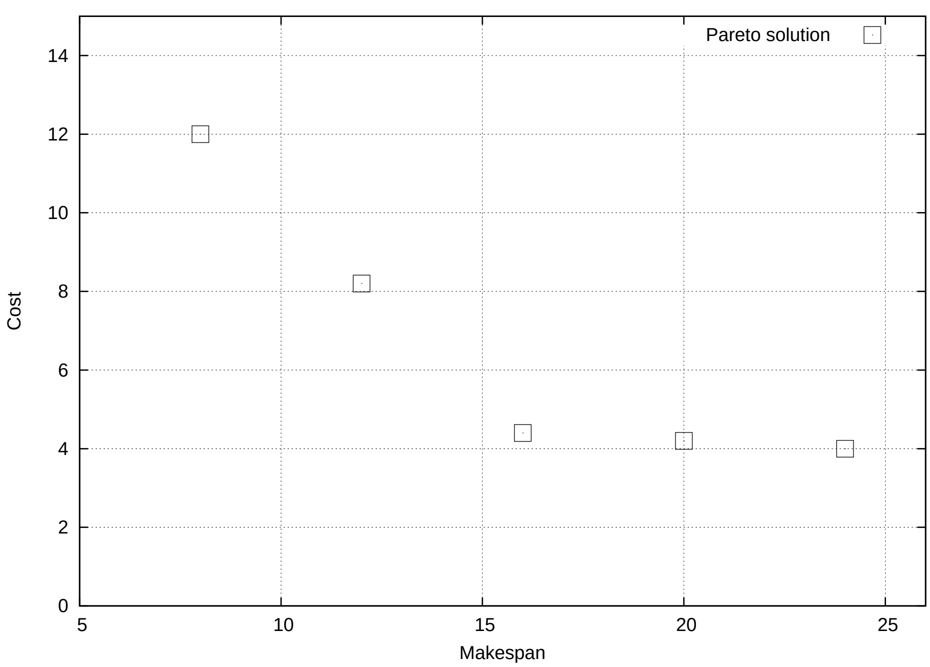
16-1.1
24-1
8-3
all-points
Attainment
Time (seconds)
1.2
1
0.8
0.6
0.4
0.2
0
0
50
100
150
200
250
300
350
400



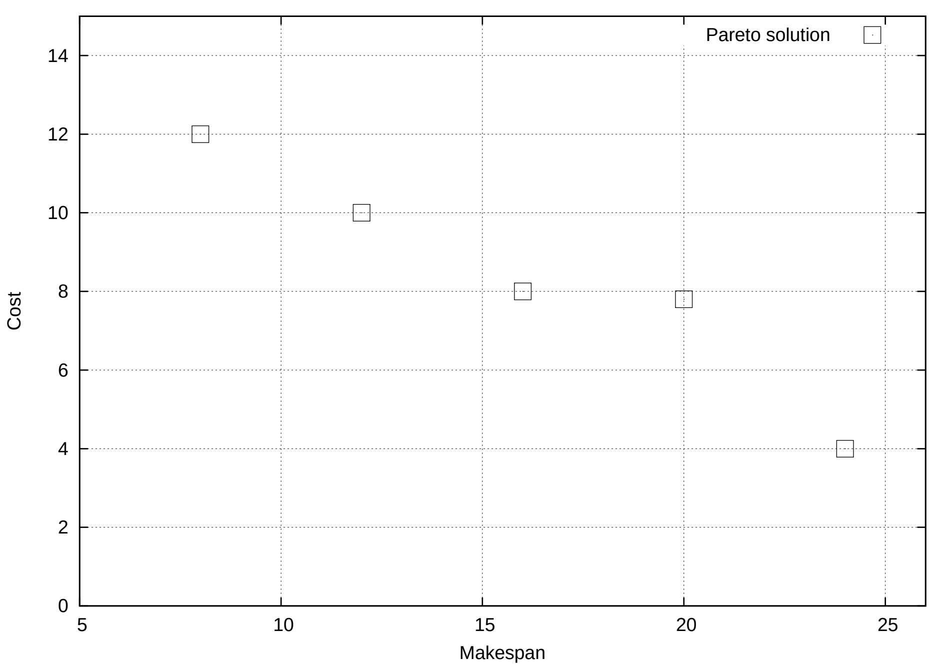
1.2
1
0.8
0.6
0.4
0.2
0
Attainment
0
50
100
150
200
250
300
350
400
Time (seconds)
12-10
16-8
20-7.8
24-4
8-12
all-points


1.2
1
0.8
0.6
0.4
0.2
0
Attainment
0
50
100
150
200
250
300
350
400
Time (seconds)
16-2.9
24-1
8-3
all-points

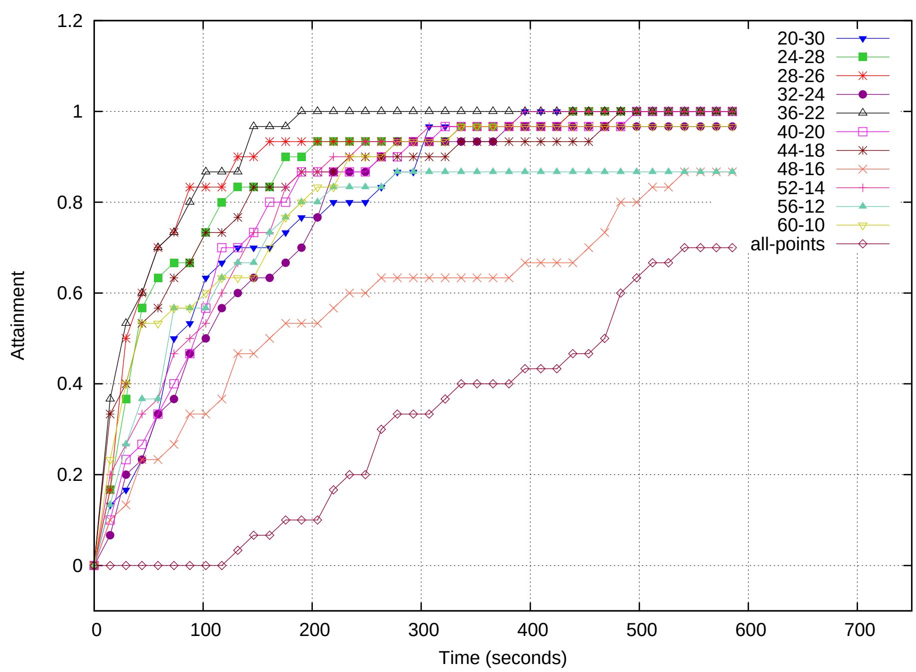

Pareto solution
Cost
Makespan
0
2
4
6
8
10
12
14
5
10
15
20
25



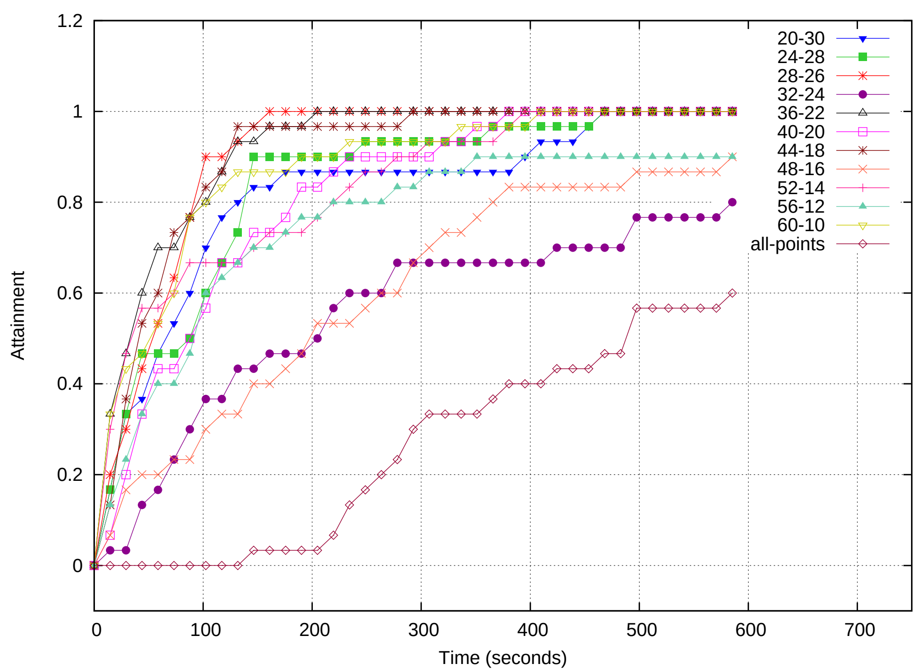
Pareto solution
14
12
10
8
6
4
2
0
Cost
5
10
15
20
25
Makespan



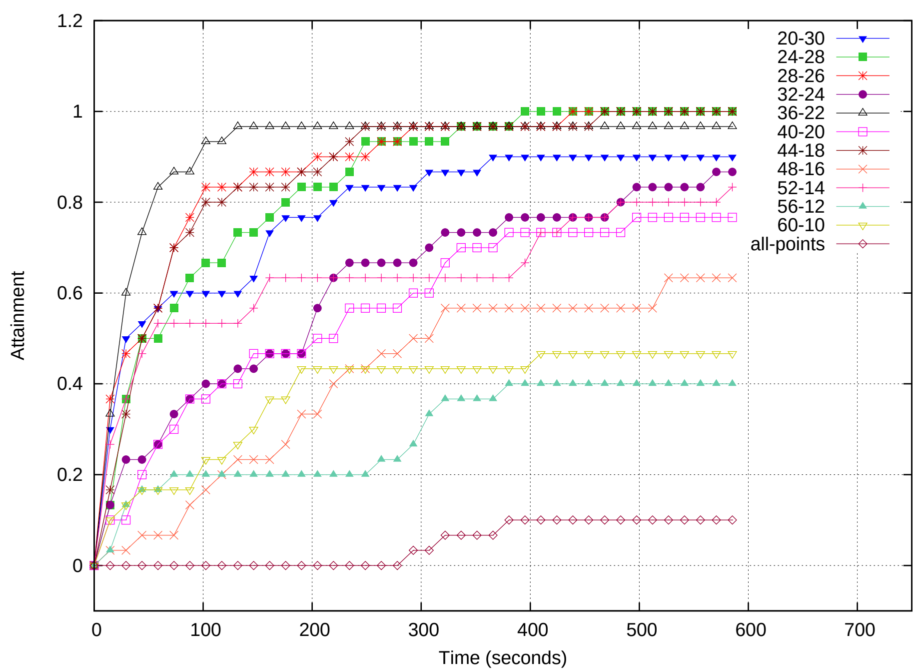
Pareto solution
Cost
Makespan
0
2
4
6
8
10
12
14
5
10
15
20
25



Attainment
Time (seconds)
1.2
1
0.8
0.6
0.4
0.2
0
0
100
200
300
400
500
600
700
20-30
24-28
28-26
32-24
36-22
40-20
44-18
48-16
52-14
56-12
60-10
all-points



1.2
1
0.8
0.6
0.4
0.2
0
Attainment
0
100
200
300
400
500
600
700
Time (seconds)
20-30
24-28
28-26
32-24
36-22
40-20
44-18
48-16
52-14
56-12
60-10
all-points



Attainment
Time (seconds)
1.2
1
0.8
0.6
0.4
0.2
0
0
100
200
300
400
500
600
700
20-30
24-28
28-26
32-24
36-22
40-20
44-18
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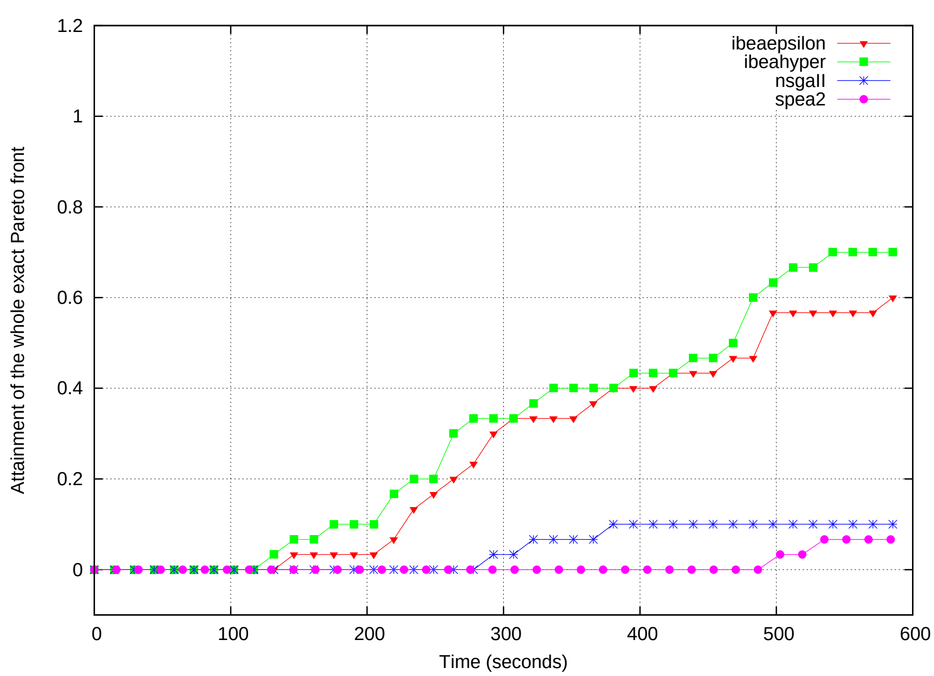
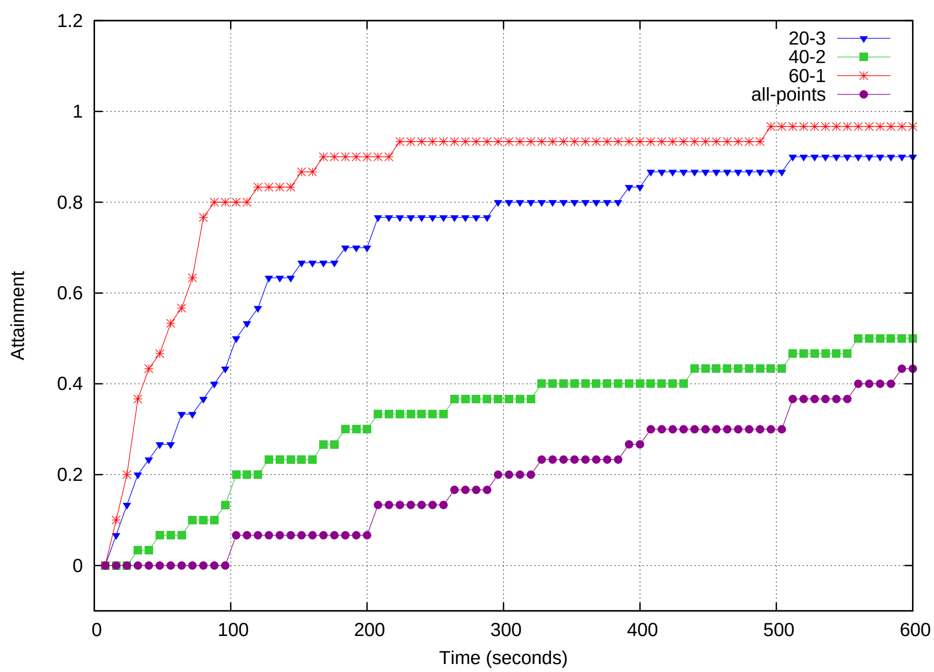
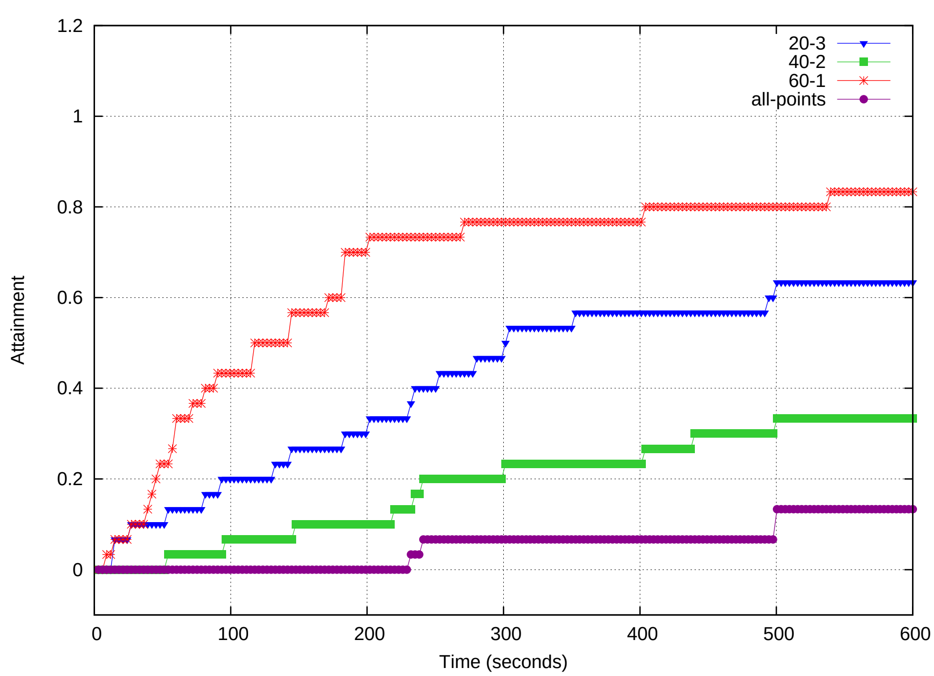
56-12
60-10
all-points



ibeaepsilon
ibeahyper
nsgaII
spea2
Attainment of the whole exact Pareto front
Time (seconds)
1.2
1
0.8
0.6
0.4
0.2
0
0
100
200
300
400
500
600



1.2
1
0.8
0.6
0.4
0.2
0
Attainment
0
100
200
300
400
500
600
Time (seconds)
20-3
40-2
60-1
all-points



1.2
1
0.8
0.6
0.4
0.2
0
Attainment
0
100
200
300
400
500
600
Time (seconds)
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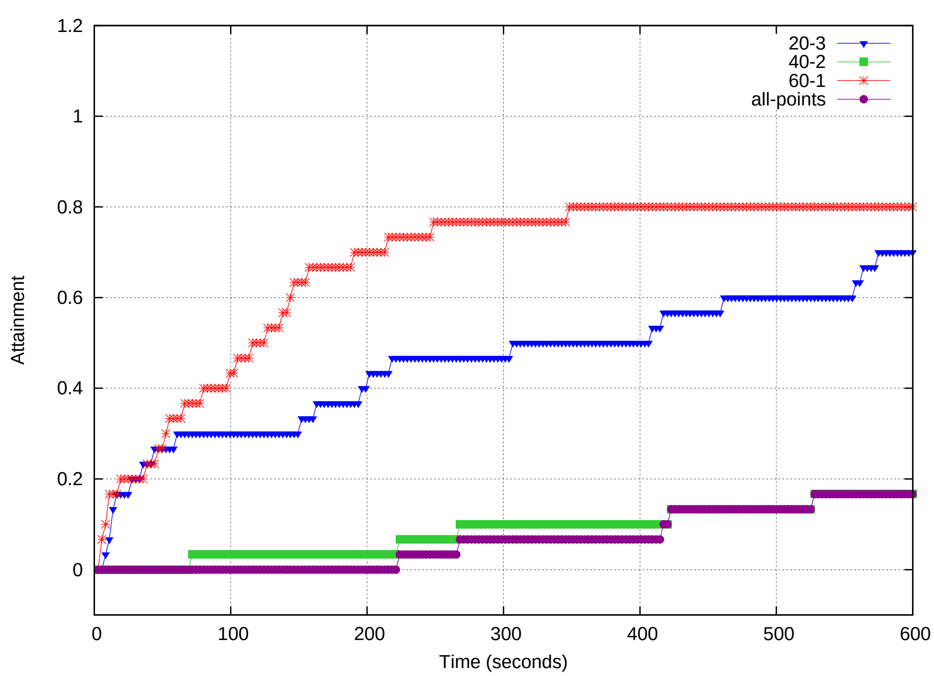
40-2
60-1
all-points


20-3
40-2
60-1
all-points
Attainment
Time (seconds)
1.2
1
0.8
0.6
0.4
0.2
0
0
100
200
300
400
500
600

Multi-Objective AI Planning: Evaluating DaE$_{\text{YAHSP}}$ on a Tunable Benchmark

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[2] ONERA-DCSD, Toulouse, France
Vincent.Vidal@onera.fr
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Abstract. All standard Artifical Intelligence (AI) planners to-date can only handle a single objective, and the only way for them to take into account multiple objectives is by aggregation of the objectives. Furthermore, and in deep contrast with the single objective case, there exists no benchmark problems on which to test the algorithms for multi-objective planning.

Divide-and-Evolve (DaE) is an evolutionary planner that won the (single-objective) deterministic temporal satisficing track in the last International Planning Competition. Even though it uses intensively the classical (and hence single-objective) planner YAHSP (*Yet Another Heuristic Search Planner*), it is possible to turn DaE$_{\text{YAHSP}}$ into a multi-objective evolutionary planner.

A tunable benchmark suite for multi-objective planning is first proposed, and the performances of several variants of multi-objective DaE$_{\text{YAHSP}}$ are compared on different instances of this benchmark, hopefully paving the road to further multi-objective competitions in AI planning.

1 Introduction

An AI Planning problem (see e.g. [1]) is defined by a set of predicates, a set of actions, an initial state and a goal state. A state is a set of non-exclusive instantiated predicates, or (Boolean) atoms. An action is defined by a set of *pre-conditions* and a set of *effects*: the action can be executed only if all pre-conditions are true in the current state, and after an action has been executed, the effects of the action modify the state: the system enters a new state. A plan in AI Planning is a sequence of actions that transforms the initial state into the goal state. The goal of AI Planning is to find a plan that minimizes some quantity related to the actions: number of actions, or sum of action costs in case actions have different costs, or makespan in the case of temporal planning, when actions have a duration and can eventually be executed in parallel. All these problems are P-SPACE.

This work was partially funded by DESCARWIN ANR project (ANR-09-COSI-002).

A simple planning problem in the domain of logistics is given in Figure 1: the problem involves cities, passengers, and planes. Passengers can be transported from one city to another, following the links on the figure. One plane can only carry one passenger at a time from one city to another, and the flight duration (number on the link) is the same whether or not the plane carries a passenger (this defines the *domain* of the problem). In the simplest non-trivial *instance* of such domain, there are 3 passengers and 2 planes. In the initial state, all passengers and planes are in `city 0`, and in the goal state, all passengers must be in `city 4`. The not-so-obvious optimal solution has a total makespan of 8 and is left as a teaser for the reader.

AI Planning is a very active field of research, as witnessed by the success of the ICAPS conferences (`http://icaps-conferences.org`), and its Intenational Planning Comptetition (IPC), where the best planners in the world compete on a set of problems. This competition has lead the researchers to design a common language to describe planning problems, PDDL (Planning Domain Definition Language). Two main categories of planners can be distinguished: *exact planners* are guaranteed to find the optimal solution ... if given enough time; *satisficing planners* give the best possible solution, but with no optimality guarantee. A complete description of the state-of-the-art planners is far beyond the scope of this paper.

However, to the best of our knowledge, all existing planners are single objective (i.e. optimize one criterion, the number of actions, the cost, or makespan, depending on the type of problem), whereas most real-world problems are in fact multi-objective and involve several contradictory objectives that need to be optimized simultaneously. For instance, in logistics, the decision maker must generally find a trade-off between duration and cost (or/and risk).

An obvious solution is to aggregate the different objectives into a single objective, generally a fixed linear combination of all objectives. Early work in that area used some twist in PDDL 2.0 [2,3,4]. PDDL 3.0, on the other hand, explicitly offered hooks for several objectives x, and a new track of IPC was dedicated to aggregated multiple objectives: the "net-benefit" track took place in 2006 [5] and 2008 [6], ... but was canceled in 2011 because of the small number of entries. In any case, no truly multi-objective approach to multi-objective planning has been proposed since the very preliminary proof-of-concept in the first *Divide-and-Evolve* paper [7].

One goal of this paper is to build on this preliminary work, and to discuss various issues related to the challenge of solving multi-objective problems with an evolutionary algorithm that is heavily based on a single-objective planner (YAHSP [8]) – and in particular to compare different state-of-the-art multi-objective evolutionary schemes when used within DaE$_{\text{YAHSP}}$. However, experimental comparison requires benchmark problems. Whereas the IPC have validated a large set of benchmark domains, with several instances of increasing complexity in each domain, nothing yet exists for multi-objective planning. The other goal of this paper is to propose a tunable set of benchmark instances, based on a simplified model of the IPC logistics domain Zeno illustrated in Fig. 1. One

advantage of this multi-objective benchmark is that the exact Pareto Front is known, at least for its simplest instances.

The paper is organized as follows: Section 2 rapidly introduces *Divide-and-Evolve*, more precisely the representation and variation operators that have been used in the single-objective version of DAE$_{\text{YAHSP}}$ that won the temporal deterministic satisficing track at the last IPC in 2011. Section 4 details the proposed benchmark, called MultiZeno, and gives hints about how to generate instances of different complexities within this framework. Section 3.2 rapidly introduces the 4 variants of multi-objective schemes that will be experimentally compared on some of the simplest instances of the MultiZeno benchmark and results of different series of experiments are discussed in Section 6. Section 7 concludes the paper, giving hints about further research directions.

2 Divide-and-Evolve

Let $\mathcal{P}_D(I, G)$ denote the planning problem defined on domain D (the predicates, the objects, and the actions), with initial state I and goal state G. In STRIPS representation model [9], a state is a list of Boolean atoms defined using the predicates of the domain, instantiated with the domain objects.

In order to solve $\mathcal{P}_D(I, G)$, the basic idea of DAE$_{\text{X}}$ is to find a sequence of states $S_1, \ldots, S_n$, and to use some embedded planner X to solve the series of planning problems $\mathcal{P}_D(S_k, S_{k+1})$, for $k \in [0, n]$ (with the convention that $S_0 = I$ and $S_{n+1} = G$). The generation and optimization of the sequence of states $(S_i)_{i \in [1,n]}$ is driven by an evolutionary algorithm. After each of the sub-problems $\mathcal{P}_D(S_k, S_{k+1})$ has been solved by the embedded planner, the concatenation of the corresponding plans (possibly compressed to take into account possible parallelism in the case of temporal planning) is a solution of the initial problem. In case one sub-problem cannot be solved by the embedded solver, the individual is said *unfeasible* and its fitness is highly penalized in order to ensure that feasible individuals always have a better fitness than unfeasible ones, and are selected only when there are not enough feasible individual. A thorough description of DAE$_{\text{X}}$ can be found in [10]. The following rest of this section will focus on the evolutionary parts of DAE$_{\text{X}}$.

2.1 Representation and Initialization

An individual in DAE$_{\text{X}}$ is hence a variable-length list of states of the given domain. However, the size of the space of lists of complete states rapidly becomes untractable when the number of objects increases. Moreover, goals of planning problems need only to be defined as partial states, involving a subset of the objects, and the aim is to find a state such that all atoms of the goal state are true. An individual in DAE$_{\text{X}}$ is thus a variable-length list of partial states, and a partial state is a variable-length list of atoms.

Previous work with DAE$_{\text{X}}$ on different domains of planning problems from the IPC benchmark series have demonstrated the need for a very careful choice

of the atoms that are used to build the partial states [11]. The method that is used today to build the partial states is based on a heuristic estimation, for each atom, of the earliest time from which it can become true [12]. These earliest start times are then used in order to restrict the candidate atoms for each partial state: the number of states is uniformly drawn between 1 and the number of estimated start times; For every chosen time, the number of atoms per state is uniformly chosen between 1 and the number of atoms of the corresponding restriction. Atoms are then added one by one: an atom is uniformly drawn in the allowed set of atoms (based on earliest possible start time), and added to the individual if it is not mutually exclusive (in short, *mutex*) with any other atom that is already there. Note that only an approximation of the complete mutex relation between atoms is known from the description of the problem, and the remaining mutexes will simply be gradually eliminated by selection, because they make the resulting individual unfeasible.

To summarize, an individual in DAE$_X$ is represented by a variable-length time-consistent sequence of partial states, and each partial state is a variable-length list of atoms that are not pairwise mutex.

2.2 Variation Operators

Crossover and mutation operators are defined on the DAE$_X$ representation in a straightforward manner - though constrained by the heuristic chronology and the partial mutex relation between atoms.

A simple one-point crossover is used, adapted to variable-length representation: both crossover points are independently chosen, uniformly in both parents. However, only one offspring is kept, the one that respects the approximate chronological constraint on the successive states. The crossover operator is applied with a population-level crossover probability.

Four different mutation operators are included: first, a population-level mutation probability is used; one an individual has been designated for mutation, the choice between the four mutation operators is made according to user-defined relative weights. The four possible mutations operate either at the individual level, by adding (addState) or removing (delState) a state, or at the state level by adding (addAtom) or removing (delAtom) some atoms in a uniformly chose state.

All mutation operators maintain the approximate chronology between the intermediate states (i.e., when adding a state, or an atom in a state), and the local consistency within all states (i.e. avoid pairwise mutexes).

2.3 Hybridization

DAE$_X$ uses an external embedded planner to solve the sequence of sub-problems defined by the ordered list of partial states. Any existing planner can in theory be used. However, there is no need for an optimality guarantee when solving the intermediate problems in order for DAE$_X$ to obtain good quality results [10]. Hence, and because several calls to this embedded planner are necessary for a

single fitness evaluation, a sub-optimal but fast planner is used: YAHSP [8] is a lookahead strategy planning system for sub-optimal planning which uses the actions in the relaxed plan to compute reachable states in order to speed up the search process.

For any given k, if the chosen embedded planner succeeds in solving $P_D(S_k, S_{k+1})$, the final complete state is computed by executing the solution plan from S_k, and becomes the initial state of the next problem. If all the sub-problems are solved by the embedded planner, the individual is called *feasible*, and the concatenation of the plans for all sub-problems is a global solution plan for $P_D(S_0 = I, S_{n+1} = G)$. However, this plan can in general be further optimized by rescheduling some of its actions, in a step called compression. The computation of all objective values is done from the compressed plan of the given individual. Finally, because the rationale for DAE_X is that all sub-problems should hopefully be easier than the initial global problem, and for computational performance reason, the search capabilities of the embedded planner YAHSP are limited by setting a maximal number of nodes that it is allowed to expand to solve any of the sub-problems (see again [10] for more details).

3 Multi-Objective Divide-and-Evolve

In some sense, the multi-objectivization of DAE_X is straightforward – as it is for most evolutionary algorithms. The "only" parts of the algorithm that require some modification are the selection parts, be it the parental selection, that chooses which individual from the population are allowed to breed, and the environmental selection (aka replacement), that decides which individuals among parents and offspring will survive to the next generation. Several schemes have been proposed in the EMOA literature (see e.g. Section 3.2), and the end of this Section will briefly introduce the ones that have been used in this work. However, a prerequisite is that all objectives are evaluated for all potential solutions, and the challenge here is that the embedded planner YAHSP performs its search based on only one objective.

3.1 Multi-objectivization Strategies

Even though YAHSP (like all known planners to-date) only solves planning problems based on one objective. However, it is possible since PDDL 3.0 to add some other quantities (aka Soft Constraints or Preferences [13]) that are simply computed throughout the execution of the final plan, without interfering with the search.

The very first proof-of-concept of multi-objective DAE_X [7], though using an exact planner in lieu of the satisficing planner YAHSP, implemented the simplest idea with respect to the second objective: ignore it (though computing its value for all individuals) at the level of the embedded planner, and let the evolutionary multi-objective take care of it. However, though YAHSP can only handle one objective at a time, it can handle either one in turn, provided they

are both defined in the PDDL domain definition file. Hence a whole bunch of smarter strategies become possible, depending on which objective YAHSP is asked to optimize every time it runs on a sub-problem. Beyond the fixed strategies, in which YAHSP always uses the same objective throughout $\text{DAE}_{\text{YAHSP}}$ runs, a simple dynamic randomized strategy has been used in this work: Once the planner is called for a given individual, the choice of which strategy to apply is made according to roulette-wheel selection based on user-defined relative weights; In the end, it will return the values of both objectives. It is hoped that the evolutionary algorithm will find a sequential partitioning of the problem that will nevertheless allow the global minimization of both objectives. Section 6.2 will experimentally compare the fixed strategies and the dynamic randomized strategy where the objective that YAHSP uses is chosen with equal probability among both objectives.

Other possible strategies include adaptive strategies, where each individual, or even each intermediate state in every individual, would carry a strategy parameter telling YAHSP which strategy to use – and this strategy parameter would be subject to mutation, too. This is left for further work.

3.2 Evolutionary Multi-Objective Schemes

Several Multi-Objective EAs (MOEAs) have been proposed in the recent years, and this work is concerned with comparing some of the most popular ones when used within the multi-objective version of $\text{DAE}_{\text{YAHSP}}$. More precisely, the following selection/reproduction schemescan be applied to any representation, and will be experimented with here: NSGA-II [14], SPEA2 [15], and IBEA [16]. They will now be quickly introduced in turn.

The **Non-dominated Sorting Genetic Algorithm** (NSGA-II) has been proposed by Deb et *al.* [14]. At each generation, the solutions contained in the current population are ranked into successive Pareto fronts in the objective space. Individuals mapping to vectors from the first front all belong to the best efficient set; individuals mapping to vectors from the second front all belong to the second best efficient set; and so on. Two values are then assigned for every solution of the population. The first one corresponds to the rank of the Pareto front the corresponding solution belongs to, and represents the quality of the solution in terms of convergence. The second one, the crowding distance, consists in estimating the density of solutions surrounding a particular point in the objective space, and represents the quality of the solution in terms of diversity. A solution is said to be better than another solution if it has a better rank value, or in case of equality, if it has a larger crowding distance.

The **Strength Pareto Evolutionary Algorithm** (SPEA) [17], introduces an improved fitness assignment strategy. It intrinsically handles an internal fixed-size archive that is used during the selection step to create offspring solutions. At a given iteration of the algorithm, each population and archive member x is assigned a strength value $S(x)$ representing the number of solutions it dominates. Then, the fitness value $F(x)$ of solution x is calculated by summing the

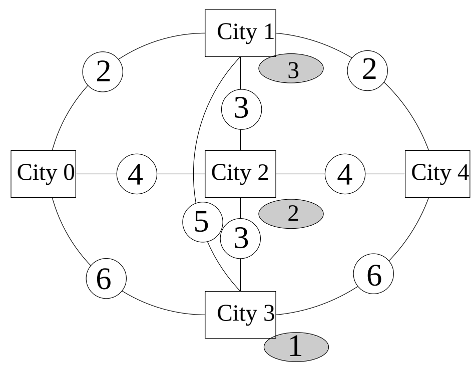


Fig. 1: A schematic view of MultiZeno, a simple benchmark transportation problem: Durations of available flights are attached to the corresponding edges, costs/risks are attached to landing in the central cities (in grey circles).

strength values of all individuals that x currently dominates. Additionally, a diversity preservation strategy is used, based on a nearest neighbor technique. The selection step consists of a binary tournament with replacement applied on the internal archive only. Last, given that the SPEA2 archive has a fixed size storage capacity, a pruning mechanism based on fitness and diversity information is used when the non-dominated set is too large.

The **Indicator-Based Evolutionary Algorithm** (IBEA) [16] introduces a total order between solutions by means of a binary quality indicator. The fitness assignment scheme of this evolutionary algorithm is based on a pairwise comparison of solutions contained in the current population with respect to a binary quality indicator I. Each individual x is assigned a fitness value $F(x)$ measuring the "loss in quality" that would result from removing x from the current population. Different indicators can be used. The most two popular, that will be used in this work, are the additive ϵ-indicator ($I_{\epsilon+}$) and the hypervolume difference indicator (I_{H-}) as defined in [16]. Each indicator $I(x, x')$ gives the minimum value by which a solution $x \in X$ can be translated in the objective space to weakly dominate another solution $x' \in X$. An archive stores solutions mapping to potentially non-dominated points in order to prevent their loss during the stochastic search process.

4 A Benchmark Suite for Multi-Objective Temporal Planning

This section details the proposed benchmark test suite for multi-objective temporal planning, based on the simple domain that is schematically described in Figure 1. The reader will have by now solved the little puzzle set in the Introduction, and found the solution with makespan 8 (flying 2 passengers to `city`

`1`, one plane continues with its passenger to `city 4` while the other plane flies back empty to `city 0`, the plane in city `city 4` returns empty to `city 1` while the other plane brings the last passenger there, and the goal is reached after both planes bring both remaining passengers to `city 4`). The rationale for this solution is that no plane ever stays idle.

In order to turn this problem into a not-too-unrealistic logistics multi-objective problem, some costs or some risks are added to all 3 central cities (1 to 3). This leads to two types of problems: In the MultiZeno$_{Cost}$, the second objective is an additive objective: each plane has to pay the corresponding tax every time it lands in that city; In the MultiZeno$_{Risk}$, the second objective is similar to a risk, and the maximal value encountered during the complete execution of a plan is to be minimized.

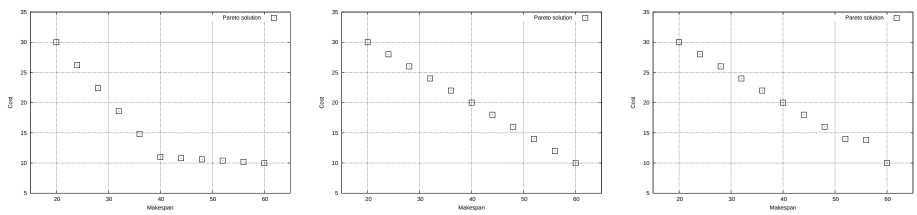
In both cases, there are 3 obvious points that belong to the Pareto Front: the solution with minimal makespan described above, and the similar solutions that use respectively `city 2` and `city 3` in lieu of `city 1`. The values of the makespans are respectively 8, 16 and 24, and the values of the costs are, for each solution, 4 times the value of the single landing tax, and exactly the value of the involved risk. For the risk case, there is no other point on the Pareto Front, as a single landing on a high-risk city sets the risk of the whole plan to a high risk. For the cost model however, there are other points on the Pareto Front, as different cities can be used for the different passengers. For instance, in the case of Figure 1, this leads to a Pareto Front made of 5 points, (8,12), (16,8), and (24,4) (going only through `city 1`, `2` and `3` respectively), plus (12,10) and (20,6). Only the first 3 are the Pareto Front in the risk case.

4.1 Tuning the Complexity

There are several ways to make this first simple instance more or less complex. A first possibility is to add passengers. In this work, only bunches of 3 passengers have been considered, in order to be able to easily derive some obvious Pareto-optimal solutions, using several times the little trick to avoid leaving any plane idle. For instance, it is easy to derive all the Pareto solutions for 6 and 9 passengers – and in the following, the corresponding instances will be termed MultiZeno3, MultiZeno6, and MultiZeno9 respectively (sub-scripted with the type of second objective – cost or risk).

Of course, the number of planes could also be increased, though the number of passengers needs to remain larger than the number of planes to allow for non-trivial Pareto front. However, departing from the 3 passengers to 2 planes ratio would make the Pareto front not easy to identify any more.

Another possibility is to increase the number of central cities: this creates more points on the Pareto front, using either plans in which a single city is used for all passengers, or plans that use several different cities for different passengers (while nevertheless using the same trick to ensure no plane stays idle). In such configuration too the exact Pareto front remains easy to identify: further work will investigate this line of complexification.



(a) $\alpha = 1.1$ (b) $\alpha = 2$ (Fig. 1) (c) $\alpha = 2.9$

Fig. 2: The exact Pareto Fronts for the MultiZeno6 problem for different values of the cost α of `city2` (those of `city1` and `city3` being 3 and 1 respectively).

4.2 Modifying the shape of the Pareto Front

Another way to change the difficulty of the problem without increasing its complexity is to tune the different values of the flight times and the cost/risk at each city. Such changes does not modify the number of points on the Pareto Front, but does change its shape in the objective space. For instance, simply modifying the cost α of `city2`, the central city in Figure 1, between 1 and 3 (the costs of respectively `city1` and `city3`), the Pareto Front, which is linear for $\alpha = 2$ becomes strictly convex for $\alpha < 2$ and strictly concave for $\alpha > 2$, as can be seen for two extreme cases ($\alpha = 1.1$ and $\alpha = 2.9$) on Figure 2. Further work will address the identification of the correct domain parameters in order to reach a given shape of the Pareto front.

5 Experimental Conditions

Implementation: All proposed multi-objective approaches (see Section 3.2) have been implemented within the ParadisEO-MOEO framework [18]. All experiments were performed on the MultiZeno3, MultiZeno6, and MultiZeno9 instances. The first objective is the makespan, and the second objective either the (additive) cost or the (maximal) risk, as discussed in Section 4. The values of the different flight durations and cost/risks are those given on Figure 1 except otherwise stated.

Parameter tuning: All user-defined parameters have been tuned using the framework ParamILS [19]. ParamILS handles any parameterized algorithm whose parameters can be discretized. Based on Iterated Local Search (ILS), ParamILS searches through the space of possible parameter configurations, evaluating configurations by running the algorithm to be optimized on a set of benchmark instances, searching for the configuration that yields overall best performance across the benchmark problems. Here, both the parameters of the multi-objective algorithms (including the internal parameters of the variation operators – see [20]) and YAHSP specific parameters (including the relative weights of the possible strategies (see Section 3.1) have been subject to ParamILS optimization.

For the purpose of this work, parameters were tuned anew for each instance (see [20] for a discussion about the generality of such parameter tuning, that falls beyond the scope of this paper).

Performance Metric: The quality measure used by ParamILS to optimize DaE$_{\text{YAHSP}}$ is the unary hypervolume I_{H^-} [16] of the set of non-dominated points output by the algorithm with respect to the complete true Pareto front (only instances where the true Pareto front is fully known have been experimented with). The lower the better (a value of 0 indicates that the exact Pareto front has been reached).

However, and because the true front is known exactly, and is made of a few scattered points (at most 17 for MultiZeno9 in this paper), it is also possible to visually monitor when each point of the front is discovered by the algorithm. This allows some deeper comparison between algorithms even when none has found the whole front. Such *attainment plots* will be used in the following, together with more classical plots of hypervolume vs time.

For all experiments, 30 independent runs were performed. Note that all the performance assessment procedures, including the hypervolume calculations, have been achieved using the PISA performance assessment tool suite [21].

Stopping Criterion: Because different fitness evaluations involve different number calls to YAHSP – and because YAHSP runs can have different computational costs too, depending on the difficulty of the sub-problem being solved – the stopping criterion was a fixed amount of CPU time rather than the usual number of fitness evaluation. These absolute limits were set to 300, 600, and 900 seconds respectively for MultiZeno3, MultiZeno6, and MultiZeno9.

6 Experimental Results

6.1 Comparing Multi-Objective Schemes

The first series of experiments presented here are concerned with the comparison of the different multi-objective schemes briefly introduced in Section 3.2. Figure 3 displays a summary of experiments of all 4 variants for MultiZeno instances for both the *Cost* and *Risk* problems.

Some clear conclusions can be drawn from these results, that are confirmed by the statistical analyses presented in Table 1 using Wilcoxon signed rank test with 95% confidence level. First, looking at the minimal values of the hypervolume reached by the different algorithms shows that, as expected, the difficulty of the problems increases with the number of passengers, and for a given complexity, the *Risk* problems are more difficult to solve than the *Cost* ones. Second, from the plots and the statistical tests, it can be seen that NSGA-II is outperformed by all other variants on all problems, SPEA2 by both indicator-based variants on most instances, and $IBEA_{H^-}$ is a clear winner over $IBEA_{\varepsilon^+}$ except on MultiZeno6$_{risk}$.

More precisely, Figure 4 show the cumulated final populations of all 30 runs in the objective space together with the true Pareto front for MultiZeno6-9$_{cost}$ problems: the situation is not as bad as it seemed from Figure 3-(e) for MultiZeno9$_{cost}$, as most solutions that are returned by $IBEA_{H^-}$ are close to the Pareto front (this is even more true on MultiZeno6$_{cost}$ problem). A dynamic view of the attainment plots is given in Figure 6-(c): two points of the Pareto front are more difficult to reach than the others, namely (48,16) and (56,12).

(a) MultiZeno3$_{cost}$

(b) MultiZeno3$_{risk}$

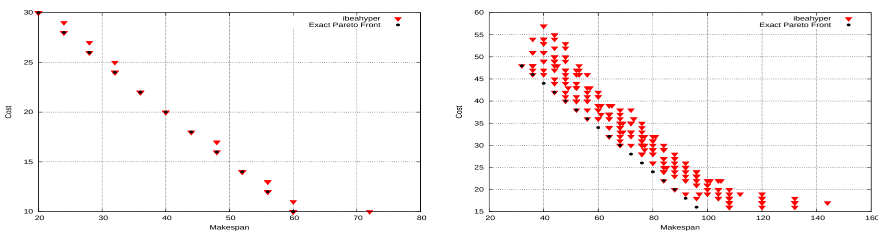
(c) MultiZeno6$_{cost}$

(d) MultiZeno6$_{risk}$

(e) MultiZeno9$_{cost}$

(f) MultiZeno9$_{risk}$

Fig. 3: Evolution of the Hypervolume indicator I_{H^-} (averaged over 30 runs) on MultiZeno instances (see Table 1 for statistical significances).



(a) MultiZeno6$_{cost}$ (b) MultiZeno9$_{cost}$

Fig. 4: Pareto fronts of IBEA$_{H^-}$ on MultiZeno instances.

(a) MultiZeno6$_{cost}$ (b) MultiZeno6$_{risk}$

Fig. 5: Attainment plots for IBEA$_{H^-}$ on MultiZeno6 instances.

6.2 Influence of YAHSP Strategy

Next series of experiments aimed at identifying the influence of the chosen strategy for YAHSP (see Section 3.1). Figure 6-(a) (resp. 6-(b)) shows the attainment plots for the strategy in which YAHSP always optimizes the makespan (resp. the cost) on problem MultiZeno6$_{cost}$. Both extreme strategies lead to much worse results than the mixed strategy of Figure 5-(a), as no run discovers the whole front (last line, that never leaves the x-axis). Furthermore, and as could be expected, the makespan-only strategy discovers very rapidly the extreme points of the Pareto front that have a small makespan (points (20,30), (24,28) and (28,26)) and hardly discovers the other end of the Pareto front (points with makespan greater than 48), while it is exactly the opposite for the cost-only strategy. This confirms the need for a strategy that incorporates both approaches. best possible choice.

Note that similar conclusion could have been drawn from ParamILS results on parameter tuning (see Section 5): the choice of YAHSP strategy was one of the parameters tuned by ParamILS . . . and the tuned values for the weights of both strategies were always more or less equal.

Table 1: Wilcoxon signed rank tests at 95% confidence level (I_{H^-} metric).

Instances	Algorithms	Algorithms			
		$NSGAII$	$IBEA_{\varepsilon+}$	$IBEA_{H-}$	$SPEA2$
$Zeno3_{cost}$	$NSGAII$	–	≡	≡	≡
	$IBEA_{\varepsilon+}$	≡	–	≡	≡
	$IBEA_{H-}$	≡	≡	–	≡
	$SPEA2$	≡	≡	≡	–
$Zeno3_{risk}$	$NSGAII$	–	≡	≡	≡
	$IBEA_{\varepsilon+}$	≡	–	≡	≻
	$IBEA_{H-}$	≡	≡	–	≻
	$SPEA2$	≡	≺	≺	–
$Zeno6_{cost}$	$NSGAII$	–	≺	≺	≺
	$IBEA_{\varepsilon+}$	≻	–	≡	≡
	$IBEA_{H-}$	≻	≡	–	≡
	$SPEA2$	≻	≡	≡	–
$Zeno6_{risk}$	$NSGAII$	–	≺	≺	≡
	$IBEA_{\varepsilon+}$	≻	–	≻	≻
	$IBEA_{H-}$	≻	≺	–	≻
	$SPEA2$	≡	≺	≺	–
$Zeno9_{cost}$	$NSGAII$	–	≺	≺	≺
	$IBEA_{\varepsilon+}$	≻	–	≺	≡
	$IBEA_{H-}$	≻	≻	–	≡
	$SPEA2$	≻	≡	≡	–
$Zeno9_{risk}$	$NSGAII$	–	≺	≺	≺
	$IBEA_{\varepsilon+}$	≻	–	≺	≡
	$IBEA_{H-}$	≻	≻	–	≡
	$SPEA2$	≻	≡	≡	–

(a) YAHSP optimizes makespan

(b) YAHSP optimizes cost

Fig. 6: Attainment plots for two search strategies on MultiZeno6$_{cost}$.

6.3 Shape of the Pareto Front

Figure 7 displays the attainment plots of IBEA$_{H^-}$ for both extreme Pareto fronts shown on Figure 2 – while the corresponding plot for the linear case $\alpha = 2$ is that of Figure 5-(a). Whereas the concave front is fully identified in 40% of the runs (right), the complete front for the strictly convex case (left) is never reached: in the latter case, the 4 most extreme points are found by 90% of the runs in less than 200 seconds, while the central points are hardly ever found. We hypothesize that the handling of YAHSP strategy regarding which objective to optimize (see Section 3.1) has a greater influence in the case of this strictly

(a) cost(city2)=1.1 (b) cost(city2)=2.9

Fig. 7: Attainment plots for different Pareto fronts for MultiZeno6$_{cost}$.

convex front than when the front is linear ($\alpha = 2$) or almost linear, even if strictly concave ($\alpha = 2.9$). In any case, no aggregation technique could ever solve the latter case, whereas it is here solved in 40% of the runs by DaE$_{\text{YAHSP}}$.

7 Conclusion and Perspectives

The contributions of this paper are twofold. Firstly, MultiZeno, an original benchmark test suite for multi-objective temporal planning, has been detailed, and several levers identified that allow to generate more or less complex instances, that have been confirmed experimentally: increasing the number of passengers obviously makes the problem more difficult; modifying the cost of reaching the cities and the duration of the flights is another way to make the problem harder, though deeper work is required to identify the consequences of each modification. Secondly, several multi-objectivization of DaE$_{\text{X}}$, an efficient evolutionary planner in the single-objective case, have been proposed.

However, even though the hypervolume-based IBEA$_{H^-}$ clearly emerged as the best choice, the experimental comparison of those variants on the MultiZeno benchmark raises more questions than it brings answers. The sparseness of the Pareto Front has been identified as a possible source for the rather poor performance of all variants for moderately large instances, particularly for the *risk* type of instances. Some smoothening of the objectives could be beneficial to tackle this issue (e.g., counting for the number of times each risk level is hit rather than simply accounting for the maximal value reached). Another direction of research is to combat the non-symmetry of the results, due to the fact that the embedded planner only optimizes one objective. Further work will investigate a self-adaptive approach to the choice of which objective to give YAHSP to optimize. Finally, the validation of the proposed multi-objective DaE$_{\text{YAHSP}}$ can only be complete after a thorough comparison with the existing aggregation approaches – though it is clear that aggregation approaches will not be able to identify the whole Pareto front in case it has some concave parts, whereas the results reported here show that DaE$_{\text{YAHSP}}$ can reasonably do it.

References

1. Ghallab, M., Nau, D., Traverso, P.: Automated Planning, Theory and Practice. Morgan Kaufmann (2004)
2. Do, M., Kambhampati, S.: SAPA: A Multi-Objective Metric Temporal Planner. J. Artif. Intell. Res. (JAIR) **20** (2003) 155–194
3. Refanidis, I., Vlahavas, I.: Multiobjective Heuristic State-Space Planning. Artificial Intelligence **145**(1) (2003) 1–32
4. Gerevini, A., Saetti, A., Serina, I.: An Approach to Efficient Planning with Numerical Fluents and Multi-Criteria Plan Quality. Artificial Intelligence **172(8-9)** (2008) 899–944
5. Chen, Y., Wah, B., Hsu, C.: Temporal Planning using Subgoal Partitioning and Resolution in SGPlan. J. of Artificial Intelligence Research **26**(1) (2006) 323–369
6. Edelkamp, S., Kissmann, P.: Optimal Symbolic Planning with Action Costs and Preferences. In: Proc. 21^{st} IJCAI. (2009) 1690–1695
7. Schoenauer, M., Savéant, P., Vidal, V.: Divide-and-Evolve: a New Memetic Scheme for Domain-Independent Temporal Planning. In Gottlieb, J., Raidl, G., eds.: Proc. 6^{th} EvoCOP, LNCS 3906, Springer (2006) 247–260
8. Vidal, V.: A Lookahead Strategy for Heuristic Search Planning. In: Proceedings of the 14^{th} ICAPS, AAAI Press (2004) 150–159
9. Fikes, R., Nilsson, N.: STRIPS: A New Approach to the Application of Theorem Proving to Problem Solving. Artificial Intelligence **1** (1971) 27–120
10. Bibaï, J., Savéant, P., Schoenauer, M., Vidal, V.: An Evolutionary Metaheuristic Based on State Decomposition for Domain-Independent Satisficing Planning. In R. Brafman et al., ed.: Proc. 20^{th} ICAPS, AAAI Press (2010) 18–25
11. Bibaï, J., Savéant, P., Schoenauer, M., Vidal, V.: On the Benefit of Sub-Optimality within the Divide-and-Evolve Scheme. In Cowling, P., Merz, P., eds.: Proc. 10^{th} EvoCOP, LNCS 6022, Springer Verlag (2010) 23–34
12. Haslum, P., Geffner, H.: Admissible Heuristics for Optimal Planning. In: Proc. AIPS-2000. (2000) 70–82
13. Gerevini, A., Long, D.: Preferences and Soft Constraints in PDDL3. In: ICAPS Workshop on Planning with Preferences and Soft Constraints. (2006) 46–53
14. Deb, K., Pratap, A., Agarwal, S., Meyarivan, T.: A fast and elitist multiobjective genetic algorithm: NSGA-II. IEEE Trans. Evol. Comp. **6**(2) (2002) 182–197
15. Zitzler, E., Laumanns, M., Thiele, L.: SPEA2: Improving the Strength Pareto Evolutionary Algorithm for Multiobjective Optimization. In: Evol. Methods Design Optim. Control Applicat. Ind. Prob. (EUROGEN). (2002) 95–100
16. Zitzler, E., Künzli, S.: Indicator-Based Selection in Multiobjective Search. In Xin Yao et al., ed.: Proc. PPSN VIII, LNCS 3242, Springer Verlag (2004) 832–842
17. Zitzler, E., Laumanns, M., Thiele, L.: SPEA2: Improving the Strength Pareto Evolutionary Algorithm. Technical report, ETH Zürich (2001)
18. Liefooghe, A., Basseur, M., Jourdan, L., Talbi, E.: ParadisEO-MOEO: A framework for evolutionary multi-objective optimization. In: Evolutionary multi-criterion optimization, Springer (2007) 386–400
19. Hutter, F., Hoos, H.H., Leyton-Brown, K., Stützle, T.: ParamILS: an automatic algorithm configuration framework. J. Artif. Intell. Res. (JAIR) **36** (2009) 267–306
20. Bibaï, J., Savéant, P., Schoenauer, M., Vidal, V.: On the Generality of Parameter Tuning in Evolutionary Planning. In: Proc 12^{th} GECCO, ACM (2010) 241–248
21. Bleuler, S., Laumanns, M., Thiele, L., Zitzler, E.: PISA a platform and programming language independent interface for search algorithms. In: Evolutionary Multi-Criterion Optimization. Volume 2632 of LNCS. Springer (2003) 494–508

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1 Introduction

Authors wishing to code their contribution with LaTeX, as well as those who have already coded with LaTeX, will be provided with a document class that will give the text the desired layout. Authors are requested to adhere strictly to these instructions; *the class file must not be changed*.

The text output area is automatically set within an area of 12.2 cm horizontally and 19.3 cm vertically.

If you are already familiar with LaTeX, then the LLNCS class should not give you any major difficulties. It will change the layout to the required LLNCS style (it will for instance define the layout of `\section`). We had to invent some extra commands, which are not provided by LaTeX (e.g. `\institute`, see also Sect. 5)

For the main body of the paper (the text) you should use the commands of the standard LaTeX "article" class. Even if you are familiar with those commands, we urge you to read this entire documentation thoroughly. It contains many suggestions on how to use our commands properly; thus your paper will be formatted exactly to LLNCS standard. For the input of the references at the end of your contribution, please follow our instructions given in Sect. 17 References.

The majority of these hints are not specific for LLNCS; they may improve your use of LaTeX in general. Furthermore, the documentation provides suggestions about the proper editing and use of the input files (capitalization, abbreviation etc.) (see Sect. 4 How to Edit Your Input File).

2 How to Proceed

The package consists of the following files:

`history.txt`	the version history of the package
`llncs.cls`	class file for LaTeX
`llncs.dem`	an example showing how to code the text
`llncs.doc`	general instructions (source of this document), `llncs.doc` means *l*atex *doc*umentation for *L*ecture *N*otes in *C*omputer *S*cience
`llncsdoc.pdf`	the documentation of the class (PDF version),
`llncs.doc`	general instructions (source of this document),
`llncsdoc.sty`	class modifications to help for the instructions
`llncs.ind`	an external (faked) author index file
`subjidx.ind`	subject index demo from the Springer book package
`llncs.dvi`	the resultig DVI file (remember to use binary transfer!)
`sprmindx.sty`	supplementary style file for MakeIndex (usage: `makeindex -s sprmindx.sty <yourfile.idx>`)

2.1 How to Invoke the LLNCS Document Class

The LLNCS class is an extension of the standard LaTeX "article" document class. Therefore you may use all "article" commands for the body of your contribution

to prepare your manuscript. LLNCS class is invoked by replacing "article" by "llncs" in the first line of your document:

```
\documentclass{llncs}
%
\begin{document}
  <Your contribution>
\end{document}
```

2.2 Contributions Already Coded with LaTeX without the LLNCS document class

If your file is already coded with LaTeX you can easily adapt it a posteriori to the LLNCS document class.

Please refrain from using any LaTeX or TeX commands that affect the layout or formatting of your document (i.e. commands like `\textheight`, `\vspace`, `\headsep` etc.). There may nevertheless be exceptional occasions on which to use some of them.

The LLNCS document class has been carefully designed to produce the right layout from your LaTeX input. If there is anything specific you would like to do and for which the style file does not provide a command, *please contact us*. Same holds for any error and bug you discover (there is however no reward for this – sorry).

3 General Rules for Coding Formulas

With mathematical formulas you may proceed as described in Sect. 3.3 of the *LaTeX User's Guide & Reference Manual* by Leslie Lamport (2nd ed. 1994), Addison-Wesley Publishing Company, Inc.

Equations are automatically numbered sequentially throughout your contribution using arabic numerals in parentheses on the right-hand side.

When you are working in math mode everything is typeset in italics. Sometimes you need to insert non-mathematical elements (e.g. words or phrases). Such insertions should be coded in roman (with `\mbox`) as illustrated in the following example:

Sample Input

```
\begin{equation}
  \left(\frac{a^{2} + b^{2}}{c^{3}} \right) = 1 \quad
  \mbox{ if } c\neq 0 \mbox{ and if } a,b,c\in \bbbr \enspace .
\end{equation}
```

Sample Output

$$\left(\frac{a^{2} + b^{2}}{c^{3}} \right) = 1 \quad \mbox{ if } c\neq 0 \mbox{ and if } a,b,c\in \mathbb{R} \enspace . \tag{1}$$

If you wish to start a new paragraph immediately after a displayed equation, insert a blank line so as to produce the required indentation. If there is no new paragraph either do not insert a blank line or code `\noindent` immediately before continuing the text.

Please punctuate a displayed equation in the same way as other ordinary text but with an `\enspace` before end punctuation.

Note that the sizes of the parentheses or other delimiter symbols used in equations should ideally match the height of the formulas being enclosed. This is automatically taken care of by the following LaTeX commands:

`\left(` or `\left[` and `\right)` or `\right]`.

3.1 Italic and Roman Type in Math Mode

a) In math mode LaTeX treats all letters as though they were mathematical or physical variables, hence they are typeset as characters of their own in italics. However, for certain components of formulas, like short texts, this would be incorrect and therefore coding in roman is required. Roman should also be used for subscripts and superscripts *in formulas* where these are merely labels and not in themselves variables, e.g. T_{eff} *not* T_{eff}, T_{K} *not* T_K (K = Kelvin), m_{e} *not* m_e (e = electron). However, do not code for roman if the sub/superscripts represent variables, e.g. $\sum_{i=1}^{n} a_i$.
b) Please ensure that *physical units* (e.g. pc, erg s^{-1} K, cm^{-3}, W m^{-2} Hz^{-1}, m kg s^{-2} A^{-2}) and *abbreviations* such as Ord, Var, GL, SL, sgn, const. are always set in roman type. To ensure this use the `\mathrm` command: `\mathrm{Hz}`. On p. 44 of the *LaTeX User's Guide & Reference Manual* by Leslie Lamport you will find the names of common mathematical functions, such as log, sin, exp, max and sup. These should be coded as `\log`, `\sin`, `\exp`, `\max`, `\sup` and will appear in roman automatically.
c) Chemical symbols and formulas should be coded for roman, e.g. Fe not Fe, $\mathrm{H_2O}$ not H_2O.
d) Familiar foreign words and phrases, e.g. et al., a priori, in situ, bremsstrahlung, eigenvalues should not be italicized.

4 How to Edit Your Input (Source) File

4.1 Headings

All words in headings should be capitalized except for conjunctions, prepositions (e.g. on, of, by, and, or, but, from, with, without, under) and definite and indefinite articles (the, a, an) unless they appear at the beginning. Formula letters must be typeset as in the text.

4.2 Capitalization and Non-capitalization

a) The following should always be capitalized:

   - Headings (see preceding Sect. 4.1)
   - Abbreviations and expressions in the text such as Fig(s)., Table(s), Sect(s)., Chap(s)., Theorem, Corollary, Definition etc. when used with numbers, e.g. Fig. 3, Table 1, Theorem 2.

   Please follow the special rules in Sect. 4.3 for referring to equations.

b) The following should *not* be capitalized:
   - The words figure(s), table(s), equation(s), theorem(s) in the text when used without an accompanying number.
   - Figure legends and table captions except for names and abbreviations.

4.3 Abbreviation of Words

a) The following *should* be abbreviated when they appear in running text *unless* they come at the beginning of a sentence: Chap., Sect., Fig.; e.g. The results are depicted in Fig. 5. Figure 9 reveals that ....
   Please note: Equations should usually be referred to solely by their number in parentheses: e.g. (14). However, when the reference comes at the beginning of a sentence, the unabbreviated word "Equation" should be used: e.g. Equation (14) is very important. However, (15) makes it clear that ....
b) If abbreviations of names or concepts are used throughout the text, they should be defined at first occurrence, e.g. Plurisubharmonic (PSH) Functions, Strong Optimization (SOPT) Problem.

5 How to Code the Beginning of Your Contribution

The title of a single contribution (it is mandatory) should be coded as follows:

```
\title{<Your contribution title>}
```

All words in titles should be capitalized except for conjunctions, prepositions (e.g. on, of, by, and, or, but, from, with, without, under) and definite and indefinite articles (the, a, an) unless they appear at the beginning. Formula letters must be typeset as in the text. Titles have no end punctuation.

If a long `\title` must be divided please use the code `\\` (for new line).

If you are to produce running heads for a specific volume the standard (of no such running heads) is overwritten with the `[runningheads]` option in the `\documentclass` line. For long titles that do not fit in the single line of the running head a warning is generated. You can specify an abbreviated title for the running head on odd pages with the command

```
\titlerunning{<Your abbreviated contribution title>}
```

There is also a possibility to change the text of the title that goes into the table of contents (that's for volume editors only – there is no table of contents for a single contribution). For this use the command

```
\toctitle{<Your changed title for the table of contents>}
```

An optional subtitle may follow then:

```
\subtitle{<subtitle of your contribution>}
```

Now the name(s) of the author(s) must be given:

```
\author{<author(s) name(s)>}
```

Numbers referring to different addresses or affiliations are to be attached to each author with the `\inst{<no>}` command. If there is more than one author, the order is up to you; the `\and` command provides for the separation.

If you have done this correctly, this entry now reads, for example:

```
\author{Ivar Ekeland\inst{1} \and Roger Temam\inst{2}}
```

The first name[1] is followed by the surname.

As for the title there exist two additional commands (again for volume editors only) for a different author list. One for the running head (on odd pages) – if there is any:

```
\authorrunning{<abbreviated author list>}
```

And one for the table of contents where the affiliation of each author is simply added in braces.

```
\tocauthor{<enhanced author list for the table of contents>}
```

Next the address(es) of institute(s), company etc. is (are) required. If there is more than one address, the entries are numbered automatically with `\and`, in the order in which you type them. Please make sure that the numbers match those placed next to to the authors' names to reflect the affiliation.

```
\institute{<name of an institute>
\and <name of the next institute>
\and <name of the next institute>}
```

In addition, you can use

```
\email{<email address>}
```

to provide your email address within `\institute`. If you need to typeset the tilde character – e.g. for your web page in your unix system's home directory – the `\homedir` command will happily do this. Please note that, if your email address is given in your paper, it will also be included in the meta data of the online version.

If footnote like things are needed anywhere in the contribution heading please code (immediately after the word where the footnote indicator should be placed):

```
\thanks{<text>}
```

[1] Other initials are optional and may be inserted if this is the usual way of writing your name, e.g. Alfred J. Holmes, E. Henry Green.

`\thanks` may only appear in `\title`, `\author` and `\institute` to footnote anything. If there are two or more footnotes or affiliation marks to a specific item separate them with `\fnmsep` (i.e. *f*oot*n*ote *m*ark *sep*arator).

The command

```
\maketitle
```

then formats the complete heading of your article. If you leave it out the work done so far will produce *no* text.

Then the abstract should follow. Simply code

```
\begin{abstract}
<Text of the summary of your article>
\end{abstract}
```

or refer to the demonstration file `llncs.dem` for an example or to the *Sample Input* on p. 12.

Remark to Running Heads and the Table of Contents

If you are the author of a single contribution you normally have no running heads and no table of contents. Both are done only by the editor of the volume or at the printers.

6 Special Commands for the Volume Editor

The volume editor can produce a complete camera ready output including running heads, a table of contents, preliminary text (frontmatter), and index or glossary. For activating the running heads there is the class option `[runningheads]`.

The table of contents of the volume is printed wherever `\tableofcontents` is placed. A simple compilation of all contributions (fields `\title` and `\author`) is done. If you wish to change this automatically produced list use the commands

```
\titlerunning  \toctitle
\authorrunning \tocauthor
```

to enhance the information in the specific contributions. See the demonstration file `llncs.dem` for examples.

An additional structure can be added to the table of contents with the `\addtocmark{<text>}` command. It has an optional numerical argument, a digit from 1 through 3. 3 (the default) makes an unnumbered chapter like entry in the table of contents. If you code `\addtocmark[2]{text}` the corresponding page number is listed also, `\addtocmark[1]{text}` even introduces a chapter number beyond it.

7 How to Code Your Text

The contribution title and all headings should be capitalized except for conjunctions, prepositions (e.g. on, of, by, and, or, but, from, with, without, under) and definite and indefinite articles (the, a, an) unless they appear at the beginning. Formula letters must be typeset as in the text.

Headings will be automatically numbered by the following codes.

Sample Input

```
\section{This is a First-Order Title}
\subsection{This is a Second-Order Title}
\subsubsection{This is a Third-Order Title.}
\paragraph{This is a Fourth-Order Title.}
```

`\section` and `\subsection` have no end punctuation.
`\subsubsection` and `\paragraph` need to be punctuated at the end.

In addition to the above-mentioned headings your text may be structured by subsections indicated by run-in headings (theorem-like environments). All the theorem-like environments are numbered automatically throughout the sections of your document – each with its own counter. If you want the theorem-like environments to use the same counter just specify the documentclass option `envcountsame`:

```
\documentclass[envcountsame]{llncs}
```

If your first call for a theorem-like environment then is e.g. `\begin{lemma}`, it will be numbered 1; if corollary follows, this will be numbered 2; if you then call lemma again, this will be numbered 3.

But in case you want to reset such counters to 1 in each section, please specify the documentclass option `envcountreset`:

```
\documentclass[envcountreset]{llncs}
```

Even a numbering on section level (including the section counter) is possible with the documentclass option `envcountsect`.

8 Predefined Theorem like Environments

The following variety of run-in headings are at your disposal:

a) **Bold** run-in headings with italicized text as built-in environments:

```
\begin{corollary} <text> \end{corollary}
\begin{lemma} <text> \end{lemma}
\begin{proposition} <text> \end{proposition}
\begin{theorem} <text> \end{theorem}
```

b) The following generally appears as *italic* run-in heading:

```
\begin{proof} <text>    \qed    \end{proof}
```

It is unnumbered and may contain an eye catching square (call for that with \qed) before the environment ends.

c) Further *italic* or **bold** run-in headings with roman environment body may also occur:

```
\begin{definition} <text> \end{definition}
\begin{example} <text> \end{example}
\begin{exercise} <text> \end{exercise}
\begin{note} <text> \end{note}
\begin{problem} <text> \end{problem}
\begin{question} <text> \end{question}
\begin{remark} <text> \end{remark}
\begin{solution} <text> \end{solution}
```

9 Defining your Own Theorem like Environments

We have enhanced the standard \newtheorem command and slightly changed its syntax to get two new commands \spnewtheorem and \spnewtheorem* that now can be used to define additional environments. They require two additional arguments namely the type style in which the keyword of the environment appears and second the style for the text of your new environment.

\spnewtheorem can be used in two ways.

9.1 Method 1 *(preferred)*

You may want to create an environment that shares its counter with another environment, say *main theorem* to be numbered like the predefined *theorem*. In this case, use the syntax

```
\spnewtheorem{<env_nam>}[<num_like>]{<caption>}
{<cap_font>}{<body_font>}
```

Here the environment with which the new environment should share its counter is specified with the optional argument [<num_like>].

Sample Input

```
\spnewtheorem{mainth}[theorem]{Main Theorem}{\bfseries}{\itshape}
\begin{theorem} The early bird gets the worm. \end{theorem}
\begin{mainth} The early worm gets eaten. \end{mainth}
```

Sample Output

Theorem 3. *The early bird gets the worm.*

Main Theorem 4. The early worm gets eaten.

The sharing of the default counter ([theorem]) is desired. If you omit the optional second argument of \spnewtheorem a separate counter for your new environment is used throughout your document.

9.2 Method 2 *(assumes* `[envcountsect]` *documentstyle option)*

```
\spnewtheorem{<env_nam>}{<caption>}[<within>]
{<cap_font>}{<body_font>}
```

This defines a new environment `<env_nam>` which prints the caption `<caption>` in the font `<cap_font>` and the text itself in the font `<body_font>`. The environment is numbered beginning anew with every new sectioning element you specify with the optional parameter `<within>`.

Example

```
\spnewtheorem{joke}{Joke}[subsection]{\bfseries}{\rmfamily}
```

defines a new environment called `joke` which prints the caption **Joke** in boldface and the text in roman. The jokes are numbered starting from 1 at the beginning of every subsection with the number of the subsection preceding the number of the joke e.g. 7.2.1 for the first joke in subsection 7.2.

9.3 Unnumbered Environments

If you wish to have an unnumbered environment, please use the syntax

```
\spnewtheorem*{<env_nam>}{<caption>}{<cap_font>}{<body_font>}
```

10 Program Codes

In case you want to show pieces of program code, just use the `verbatim` environment or the `verbatim` package of LaTeX. (There also exist various pretty printers for some programming languages.)

Sample Input (of a simple contribution)

```
\title{Hamiltonian Mechanics}

\author{Ivar Ekeland\inst{1} \and Roger Temam\inst{2}}

\institute{Princeton University, Princeton NJ 08544, USA
\and
Universit\'{e} de Paris-Sud,
Laboratoire d'Analyse Num\'{e}rique, B\^{a}timent 425,\\
F-91405 Orsay Cedex, France}

\maketitle
%
\begin{abstract}
```

```
This paragraph shall summarize the contents of the paper
in short terms.
\end{abstract}
%
\section{Fixed-Period Problems: The Sublinear Case}
%
With this chapter, the preliminaries are over, and we begin the
search for periodic solutions \dots
%
\subsection{Autonomous Systems}
%
In this section we will consider the case when the Hamiltonian
$H(x)$ \dots
%
\subsubsection*{The General Case: Nontriviality.}
%
We assume that $H$ is
$\left(A_{\infty}, B_{\infty}\right)$-subqua\-dra\-tic
at infinity, for some constant \dots
%
\paragraph{Notes and Comments.}
The first results on subharmonics were \dots
%
\begin{proposition}
Assume $H'(0)=0$ and $ H(0)=0$. Set \dots
\end{proposition}
\begin{proof}[of proposition]
Condition (8) means that, for every $\delta'>\delta$, there is
some $\varepsilon>0$ such that \dots \qed
\end{proof}
%
\begin{example}[\rmfamily (External forcing)]
Consider the system \dots
\end{example}
\begin{corollary}
Assume $H$ is $C^{2}$ and
$\left(a_{\infty}, b_{\infty}\right)$-subquadratic
at infinity. Let \dots
\end{corollary}
\begin{lemma}
Assume that $H$ is $C^{2}$ on $\bbbr^{2n}\backslash \{0\}$
and that $H''(x)$ is \dots
\end{lemma}
\begin{theorem}[(Ghoussoub-Preiss)]
Let $X$ be a Banach Space and $\Phi:X\to\bbbr$ \dots
```

```
\end{theorem}
\begin{definition}
We shall say that a $C^{1}$ function $\Phi:X\to\bbbr$
satisfies \dots
\end{definition}
```

Sample Output (follows on the next page together with examples of the above run-in headings)

Hamiltonian Mechanics

Ivar Ekeland[1] and Roger Temam[2]

[1] Princeton University, Princeton NJ 08544, USA
[2] Université de Paris-Sud, Laboratoire d'Analyse Numérique, Bâtiment 425, F-91405 Orsay Cedex, France

Abstract. This paragraph shall summarize the contents of the paper in short terms.

1 Fixed-Period Problems: The Sublinear Case

With this chapter, the preliminaries are over, and we begin the search for periodic solutions …

1.1 Autonomous Systems

In this section we will consider the case when the Hamiltonian $H(x)$ …

The General Case: Nontriviality. We assume that H is (A_∞, B_∞)-subquadratic at infinity, for some constant …

Notes and Comments. The first results on subharmonics were …

Proposition 1. *Assume $H'(0) = 0$ and $H(0) = 0$. Set …*

Proof (of proposition). Condition (8) means that, for every $\delta' > \delta$, there is some $\varepsilon > 0$ such that … □

Example 1 (External forcing). Consider the system …

Corollary 1. *Assume H is C^2 and (a_∞, b_∞)-subquadratic at infinity. Let …*

Lemma 1. *Assume that H is C^2 on $\mathbb{R}^{2n} \backslash \{0\}$ and that $H''(x)$ is …*

Theorem 1 (Ghoussoub-Preiss). *Let X be a Banach Space and $\Phi : X \to \mathbb{R}$ …*

Definition 1. *We shall say that a C^1 function $\Phi : X \to \mathbb{R}$ satisfies …*

11 Fine Tuning of the Text

The following should be used to improve the readability of the text:

`\,`	a thin space, e.g. between numbers or between units and numbers; a line division will not be made following this space
`--`	en dash; two strokes, without a space at either end
`␣--␣`	en dash; two strokes, with a space at either end
`-`	hyphen; one stroke, no space at either end
`$-$`	minus, in the text *only*

Input

```
21\,$^{\circ}$C etc.,
Dr h.\,c.\,Rockefellar-Smith \dots
20,000\,km and  Prof.\,Dr Mallory \dots
1950--1985 \dots
this -- written on a computer -- is now printed
$-30$\,K \dots
```

Output

21 °C etc., Dr h. c. Rockefellar-Smith …
20,000 km and Prof. Dr Mallory …
1950–1985 …
this – written on a computer – is now printed
-30 K …

12 Special Typefaces

Normal type (roman text) need not be coded. *Italic* (`{\em <text>}` better still `\emph{<text>}`) or, if necessary, **boldface** should be used for emphasis.

`{\itshape Text}`	*Italicized Text*
`{\em Text}`	*Emphasized Text – if you would like to emphasize a* definition *within an italicized text (e.g. of a* theorem) *you should code the expression to be emphasized by* `\em`.
`{\bfseries Text}`	**Important Text**
`\vec{Symbol}`	Vectors may only appear in math mode. The default LaTeX vector symbol has been adapted[3] to LLNCS conventions. `$\vec{A \times B\cdot C}` yields $\boldsymbol{A \times B \cdot C}$ `$\vec{A}^{T} \otimes \vec{B} \otimes \vec{\hat{D}}$`yields $\boldsymbol{A}^T \otimes \boldsymbol{B} \otimes \hat{\boldsymbol{D}}$

[3] If you absolutely must revive the original LaTeX design of the vector symbol (as an arrow accent), please specify the option `[orivec]` in the `documentclass` line.

13 Footnotes

Footnotes within the text should be coded:

`\footnote{Text}`

Sample Input

Text with a footnote`\footnote{The footnote is automatically numbered.}` and text continues ...

Sample Output

Text with a footnote[4] and text continues ...

14 Lists

Please code lists as described below:

Sample Input

```
\begin{enumerate}
  \item First item
  \item Second item
  \begin{enumerate}
    \item First nested item
    \item Second nested item
  \end{enumerate}
  \item Third item
\end{enumerate}
```

Sample Output

1. First item
2. Second item
   (a) First nested item
   (b) Second nested item
3. Third item

15 Figures

Figure environments should be inserted after (not in) the paragraph in which the figure is first mentioned. They will be numbered automatically.

Preferably the images should be enclosed as PostScript files – best as EPS data using the epsfig package.

If you cannot include them into your output this way and use other techniques for a separate production, the figures (line drawings and those containing

[4] The footnote is automatically numbered.

halftone inserts as well as halftone figures) *should not be pasted into your laser-printer output*. They should be enclosed separately in camera-ready form (original artwork, glossy prints, photographs and/or slides). The lettering should be suitable for reproduction, and after a probably necessary reduction the height of capital letters should be at least 1.8 mm and not more than 2.5 mm. Check that lines and other details are uniformly black and that the lettering on figures is clearly legible.

To leave the desired amount of space for the height of your figures, please use the coding described below. As can be seen in the output, we will automatically provide 1 cm space above and below the figure, so that you should only leave the space equivalent to the size of the figure itself. Please note that "x" in the following coding stands for the actual height of the figure:

```
\begin{figure}
\vspace{x cm}
\caption[ ]{...text of caption...}          (Do type [ ])
\end{figure}
```

Sample Input

```
\begin{figure}
\vspace{2.5cm}
\caption{This is the caption of the figure displaying a white
eagle and a white horse on a snow field}
\end{figure}
```

Sample Output

Fig. 1. This is the caption of the figure displaying a white eagle and a white horse on a snow field

16 Tables

Table captions should be treated in the same way as figure legends, except that the table captions appear *above* the tables. The tables will be numbered automatically.

16.1 Tables Coded with LaTeX

Please use the following coding:

Sample Input

```
\begin{table}
\caption{Critical $N$ values}
\begin{tabular}{llllll}
\hline\noalign{\smallskip}
${\mathrm M}_\odot$ & $\beta_{0}$ & $T_{\mathrm c6}$ & $\gamma$
  & $N_{\mathrm{crit}}^{\mathrm L}$
  & $N_{\mathrm{crit}}^{\mathrm{Te}}$\\
\noalign{\smallskip}
\hline
\noalign{\smallskip}
 30 & 0.82 & 38.4 & 35.7 & 154 & 320 \\
 60 & 0.67 & 42.1 & 34.7 & 138 & 340 \\
120 & 0.52 & 45.1 & 34.0 & 124 & 370 \\
\hline
\end{tabular}
\end{table}
```

Sample Output

Table 1. Critical N values

$\mathrm{M}_\odot$	β_0	T_{c6}	γ	$N_{\mathrm{crit}}^{\mathrm{L}}$	$N_{\mathrm{crit}}^{\mathrm{Te}}$
30	0.82	38.4	35.7	154	320
60	0.67	42.1	34.7	138	340
120	0.52	45.1	34.0	124	370

Before continuing your text you need an empty line. . . .

For further information you will find a complete description of the tabular environment on p. 62 ff. and p. 204 of the *LaTeX User's Guide & Reference Manual* by Leslie Lamport.

16.2 Tables Not Coded with LaTeX

If you do not wish to code your table using LaTeX but prefer to have it reproduced separately, proceed as for figures and use the following coding:

Sample Input

```
\begin{table}
\caption{text of your caption}
\vspace{x cm}     % the actual height needed for your table
\end{table}
```

16.3 Signs and Characters

Special Signs. You may need to use special signs. The available ones are listed in the *LaTeX User's Guide & Reference Manual* by Leslie Lamport, pp. 41 ff. We have created further symbols for math mode (enclosed in $):

`\grole`	yields	$\gtrless$	`\getsto`	yields	$\leftrightarrows$
`\lid`	yields	$\leqq$	`\gid`	yields	$\geqq$

Gothic (Fraktur). If gothic letters are *necessary*, please use those of the relevant $\mathcal{AMS}$-TeX alphabet which are available using the amstex package of the American Mathematical Society.

In LaTeX only the following gothic letters are available: `$\Re$` yields $\Re$ and `$\Im$` yields $\Im$. These should *not* be used when you need gothic letters for your contribution. Use $\mathcal{AMS}$-TeX gothic as explained above. For the real and the imaginary parts of a complex number within math mode you should use instead: `$\mathrm{Re}$` (which yields Re) or `$\mathrm{Im}$` (which yields Im).

Script. For script capitals use the coding

`$\mathcal{AB}$` which yields $\mathcal{AB}$

(see p. 42 of the LaTeX book).

Special Roman. If you need other symbols than those below, you could use the blackboard bold characters of $\mathcal{AMS}$-TeX, but there might arise capacity problems in loading additional $\mathcal{AMS}$-TeX fonts. Therefore we created the blackboard bold characters listed below. Some of them are not esthetically satisfactory. This need not deter you from using them: in the final printed form they will be replaced by the well-designed MT (monotype) characters of the phototypesetting machine.

`\bbbc`	(complex numbers)	yields	$\mathbb{C}$	`\bbbf`	(blackboard bold F)	yields	$\mathbb{F}$
`\bbbh`	(blackboard bold H)	yields	$\mathbb{H}$	`\bbbk`	(blackboard bold K)	yields	$\mathbb{K}$
`\bbbm`	(blackboard bold M)	yields	$\mathbb{M}$	`\bbbn`	(natural numbers N)	yields	$\mathbb{N}$
`\bbbp`	(blackboard bold P)	yields	$\mathbb{P}$	`\bbbq`	(rational numbers)	yields	$\mathbb{Q}$
`\bbbr`	(real numbers)	yields	$\mathbb{R}$	`\bbbs`	(blackboard bold S)	yields	$\mathbb{S}$
`\bbbt`	(blackboard bold T)	yields	$\mathbb{T}$	`\bbbz`	(whole numbers)	yields	$\mathbb{Z}$
`\bbbone`	(symbol one)	yields	$\mathbb{1}$				

$$\mathbb{C}^{\mathbb{C}^{\mathbb{C}}} \otimes \mathbb{F}_{\mathbb{F}_{\mathbb{F}}} \otimes \mathbb{H}_{\mathbb{H}_{\mathbb{H}}} \otimes \mathbb{K}_{\mathbb{K}_{\mathbb{K}}} \otimes \mathbb{M}^{\mathbb{M}^{\mathbb{M}}} \otimes \mathbb{N}_{\mathbb{N}_{\mathbb{N}}} \otimes \mathbb{P}^{\mathbb{P}^{\mathbb{P}}}$$
$$\otimes \mathbb{Q}_{\mathbb{Q}_{\mathbb{Q}}} \otimes \mathbb{R}^{\mathbb{R}^{\mathbb{R}}} \otimes \mathbb{S}^{\mathbb{S}_{\mathbb{S}}} \otimes \mathbb{T}^{\mathbb{T}^{\mathbb{T}}} \otimes \mathbb{Z} \otimes \mathbb{1}^{\mathbb{1}_{\mathbb{1}}}$$

17 References

There are three reference systems available; only one, of course, should be used for your contribution. With each system (by number only, by letter-number or by author-year) a reference list containing all citations in the text, should be included at the end of your contribution placing the LaTeX environment `thebibliography` there. For an overall information on that environment see the *LaTeX User's Guide & Reference Manual* by Leslie Lamport, p. 71.

There is a special BibTeX style for LLNCS that works along with the class: `splncs.bst` – call for it with a line `\bibliographystyle{splncs}`. If you plan to use another BibTeX style you are customed to, please specify the option `[oribibl]` in the `documentclass` line, like:

```
\documentclass[oribibl]{llncs}
```

This will retain the original LaTeX code for the bibliographic environment and the `\cite` mechanism that many BibTeX applications rely on.

17.1 References by Letter-Number or by Number Only

References are cited in the text – using the `\cite` command of LaTeX – by number or by letter-number in square brackets, e.g. [1] or [E1, S2], [P1], according to your use of the `\bibitem` command in the `thebibliography` environment. The coding is as follows: if you choose your own label for the sources by giving an optional argument to the `\bibitem` command the citations in the text are marked with the label you supplied. Otherwise a simple numbering is done, which is preferred.

```
The results in this section are a refined version
of \cite{clar:eke}; the minimality result of Proposition~14
was the first of its kind.
```

The above input produces the citation: "… refined version of [CE1]; the minimality…". Then the `\bibitem` entry of the `thebibliography` environment should read:

```
\begin{thebibliography}{[MT1]}
.
.
\bibitem[CE1]{clar:eke}
Clarke, F., Ekeland, I.:
Nonlinear oscillations and boundary-value problems for
Hamiltonian systems.
Arch. Rat. Mech. Anal. {\bfseries 78} (1982) 315--333
.
.
\end{thebibliography}
```

The complete bibliography looks like this:

References

CE1. Clarke, F., Ekeland, I.: Nonlinear oscillations and boundary-value problems for Hamiltonian systems. Arch. Rat. Mech. Anal. **78** (1982) 315–333

CE2. Clarke, F., Ekeland, I.: Solutions périodiques, du période donnée, des équations hamiltoniennes. Note CRAS Paris **287** (1978) 1013–1015

MT1. Michalek, R., Tarantello, G.: Subharmonic solutions with prescribed minimal period for nonautonomous Hamiltonian systems. J. Diff. Eq. **72** (1988) 28–55

Ta1. Tarantello, G.: Subharmonic solutions for Hamiltonian systems via a $\mathbb{Z}_p$ pseudoindex theory. Annali di Matematica Pura (to appear)

Ra1. Rabinowitz, P.: On subharmonic solutions of a Hamiltonian system. Comm. Pure Appl. Math. **33** (1980) 609–633

Number-Only System. For this preferred system do not use the optional argument in the `\bibitem` command: then, only numbers will appear for the citations in the text (enclosed in square brackets) as well as for the marks in your bibliography (here the number is only end-punctuated without square brackets).

Subsequent citation numbers in the text are collapsed to ranges. Non-numeric and undefined labels are handled correctly but no sorting is done.

E.g., `\cite{n1,n3,n2,n3,n4,n5,foo,n1,n2,n3,?,n4,n5}` – where nx is the key of the x^{th} `\bibitem` command in sequence, `foo` is the key of a `\bibitem` with an optional argument, and `?` is an undefined reference – gives 1,3,2-5,foo,1-3,?,4,5 as the citation reference.

```
\begin{thebibliography}{1}
\bibitem {clar:eke}
Clarke, F., Ekeland, I.:
Nonlinear oscillations and boundary-value problems for
Hamiltonian systems.
Arch. Rat. Mech. Anal. {\bfseries 78} (1982) 315--333
\end{thebibliography}
```

17.2 Author-Year System

References are cited in the text by name and year in parentheses and should look as follows: (Smith 1970, 1980), (Ekeland et al. 1985, Theorem 2), (Jones and Jaffe 1986; Farrow 1988, Chap. 2). If the name is part of the sentence only the year may appear in parentheses, e.g. Ekeland et al. (1985, Sect. 2.1) The reference list should contain all citations occurring in the text, ordered alphabetically by surname (with initials following). If there are several works by the same author(s) the references should be listed in the appropriate order indicated below:

a) One author: list works chronologically;
b) Author and same co-author(s): list works chronologically;
c) Author and different co-authors: list works alphabetically according to co-authors.

If there are several works by the same author(s) and in the same year, but which are cited separately, they should be distinguished by the use of “a”, “b” etc., e.g. (Smith 1982a), (Ekeland et al. 1982b).

How to Code Author-Year System. If you want to use this system you have to specify the option `[citeauthoryear]` in the `documentclass`, like:

```
\documentclass[citeauthoryear]{llncs}
```

Write your citations in the text explicitly except for the year, leaving that up to LaTeX with the `\cite` command. Then give only the appropriate year as the optional argument (i.e. the label in square brackets) with the `\bibitem` command(s).

Sample Input

```
The results in this section are a refined version
of Clarke and Ekeland (\cite{clar:eke}); the minimality result of
Proposition~14 was the first of its kind.
```

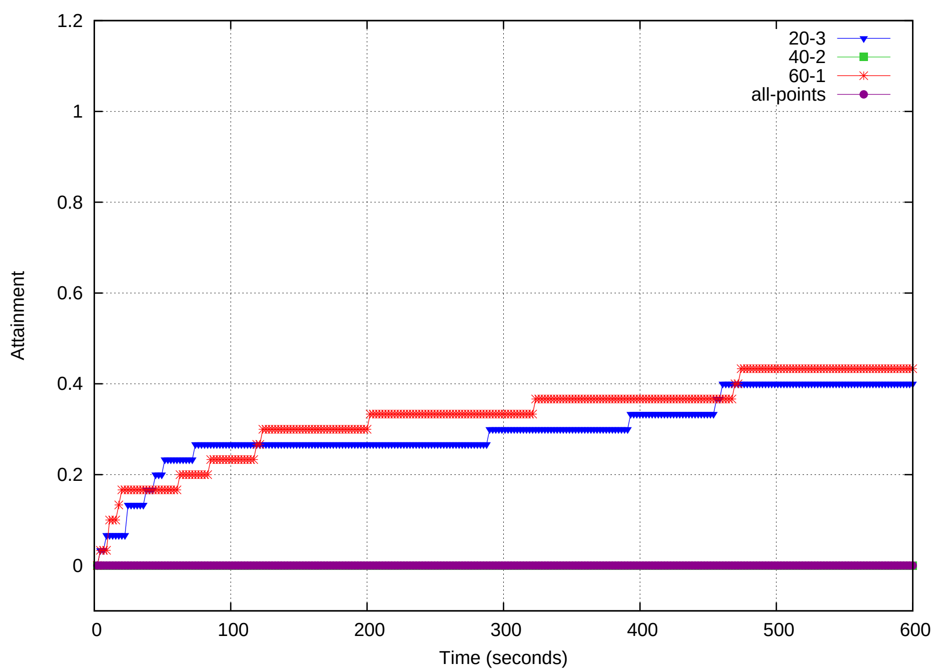
The above input produces the citation: "… refined version of Clarke and Ekeland (1982); the minimality…". Then the `\bibitem` entry of `clar:eke` in the `thebibliography` environment should read:

```
\begin{thebibliography}{}  % (do not forget {})
.
.
\bibitem[1982]{clar:eke}
Clarke, F., Ekeland, I.:
Nonlinear oscillations and boundary-value problems for
Hamiltonian systems.
Arch. Rat. Mech. Anal. {\bfseries 78} (1982) 315--333
.
.
\end{thebibliography}
```

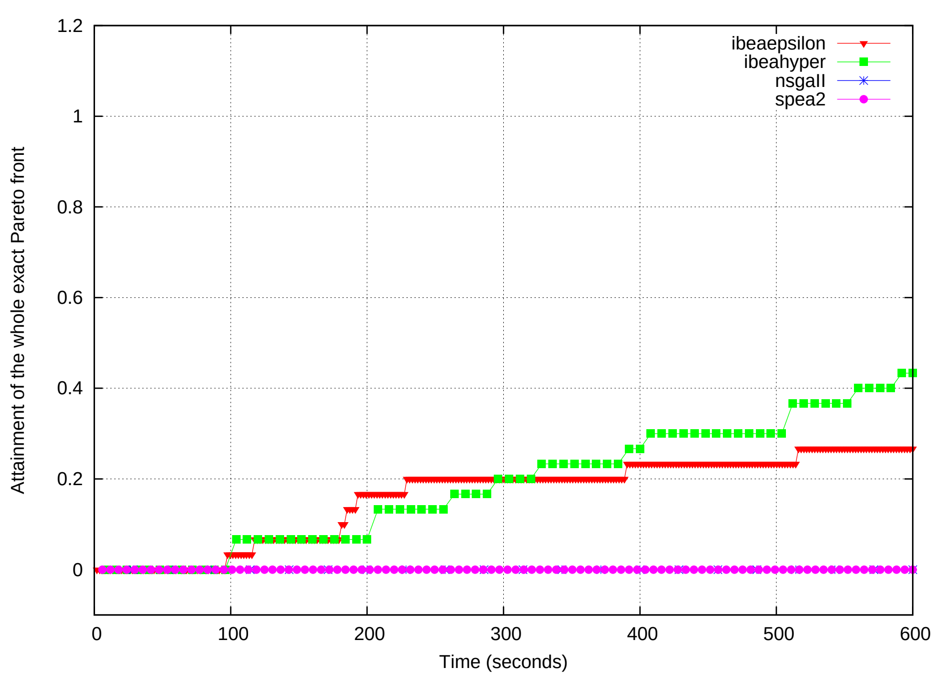
Sample Output

References

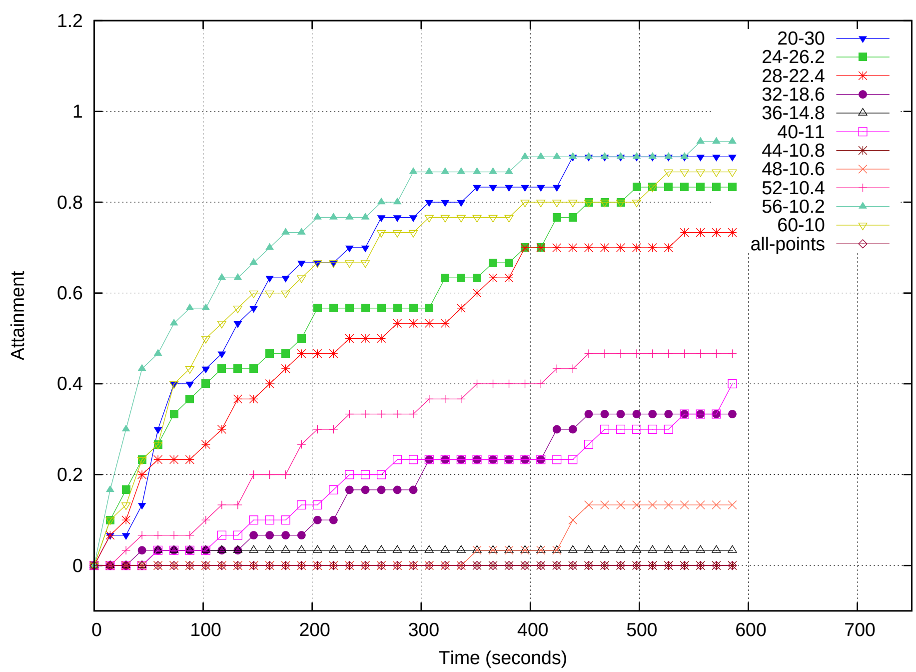
Clarke, F., Ekeland, I.: Nonlinear oscillations and boundary-value problems for Hamiltonian systems. Arch. Rat. Mech. Anal. **78** (1982) 315–333



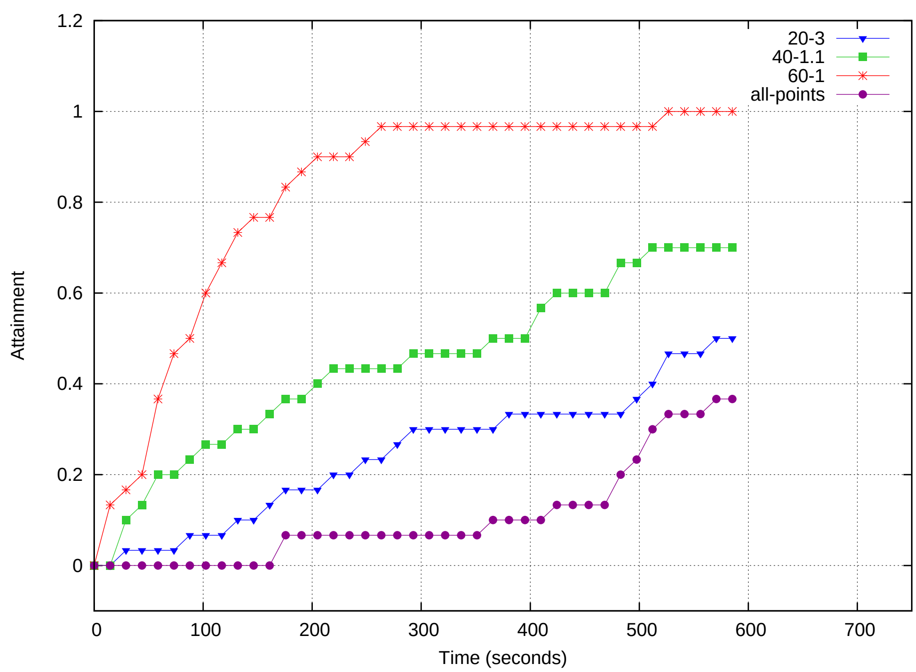
20-3
40-2
60-1
all-points
Attainment
Time (seconds)



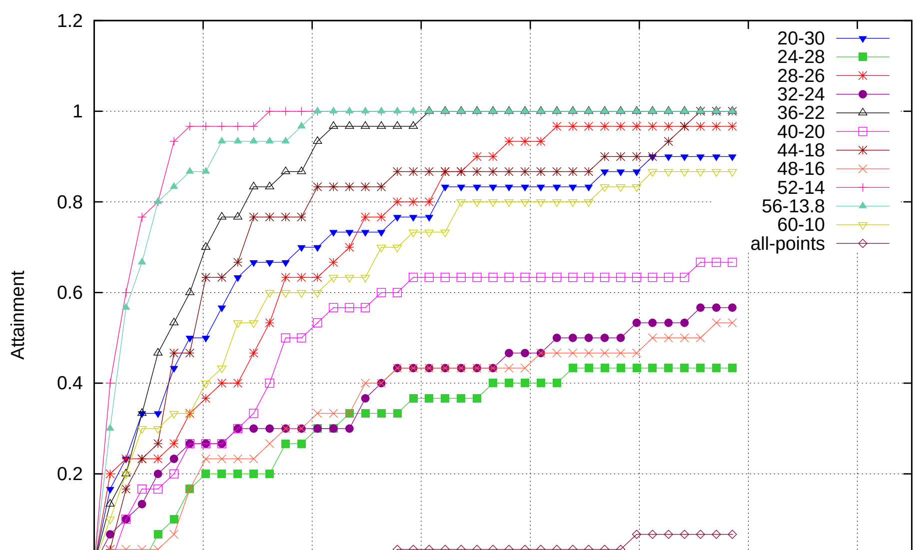
ibeaepsilon
ibeahyper
nsgaII
spea2
Attainment of the whole exact Pareto front
Time (seconds)
1.2
1
0.8
0.6
0.4
0.2
0
0
100
200
300
400
500
600



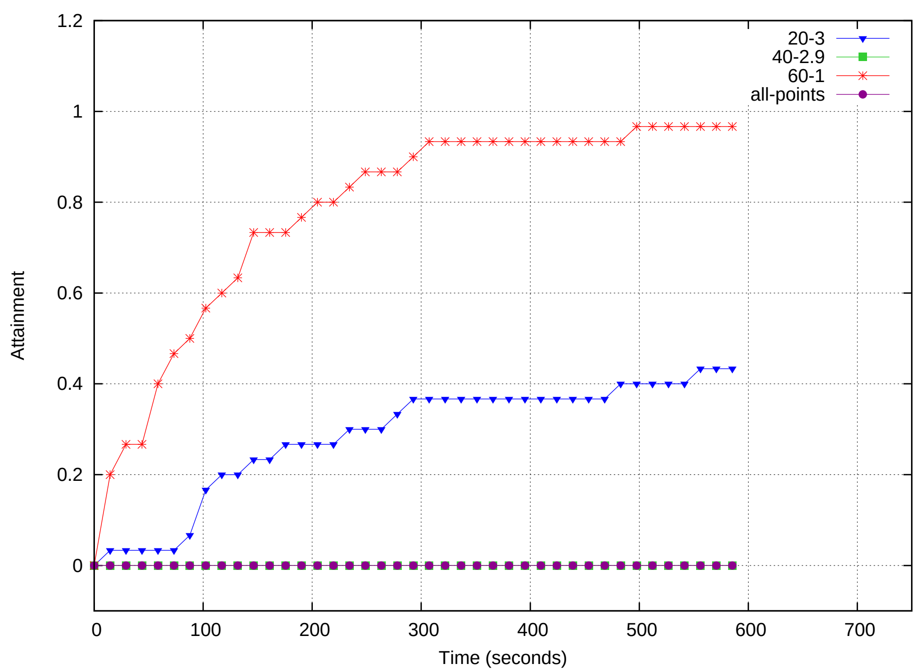
1.2
1
0.8
0.6
0.4
0.2
0
0
100
200
300
400
500
600
700
Attainment
Time (seconds)
20-30
24-26.2
28-22.4
32-18.6
36-14.8
40-11
44-10.8
48-10.6
52-10.4
56-10.2
60-10
all-points



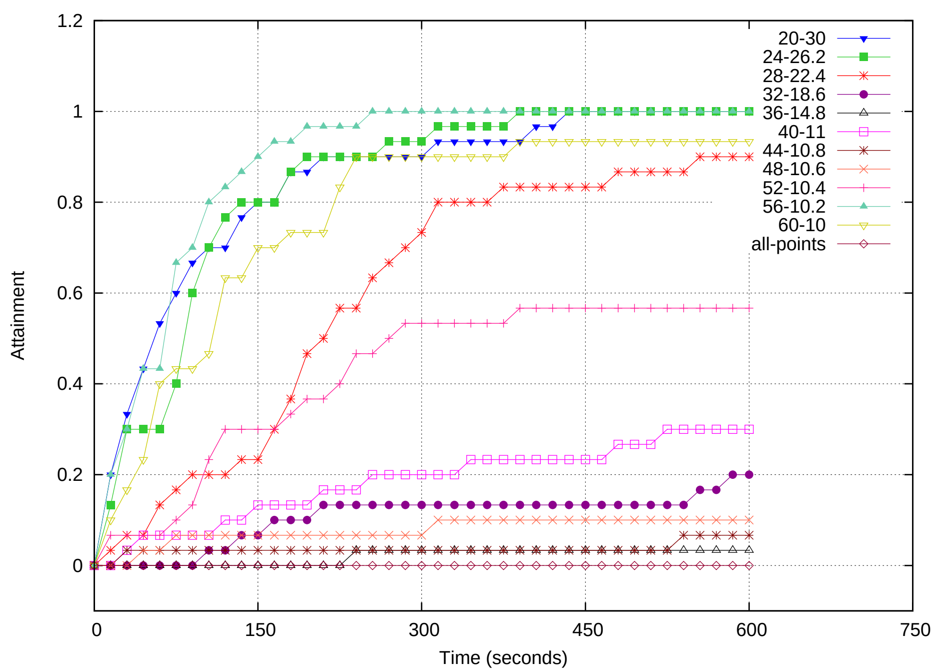
1.2
1
0.8
0.6
0.4
0.2
0
Attainment
0
100
200
300
400
500
600
700
Time (seconds)
20-3
40-1.1
60-1
all-points



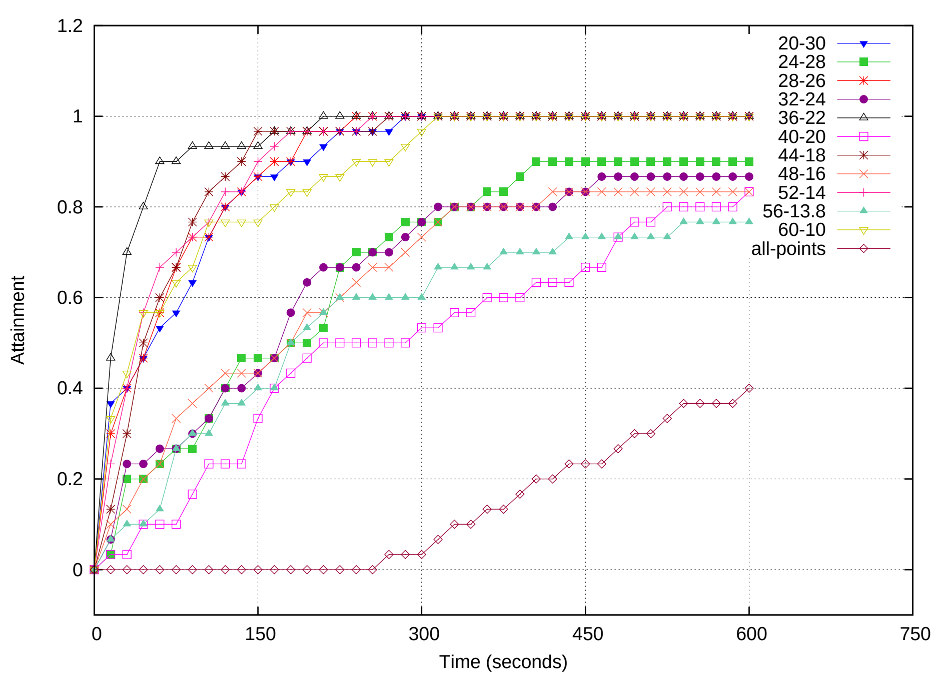
1.2
1
0.8
0.6
0.4
0.2
0
Attainment
0
100
200
300
400
500
600
700
Time (seconds)
20-30
24-28
28-26
32-24
36-22
40-20
44-18
48-16
52-14
56-13.8
60-10
all-points



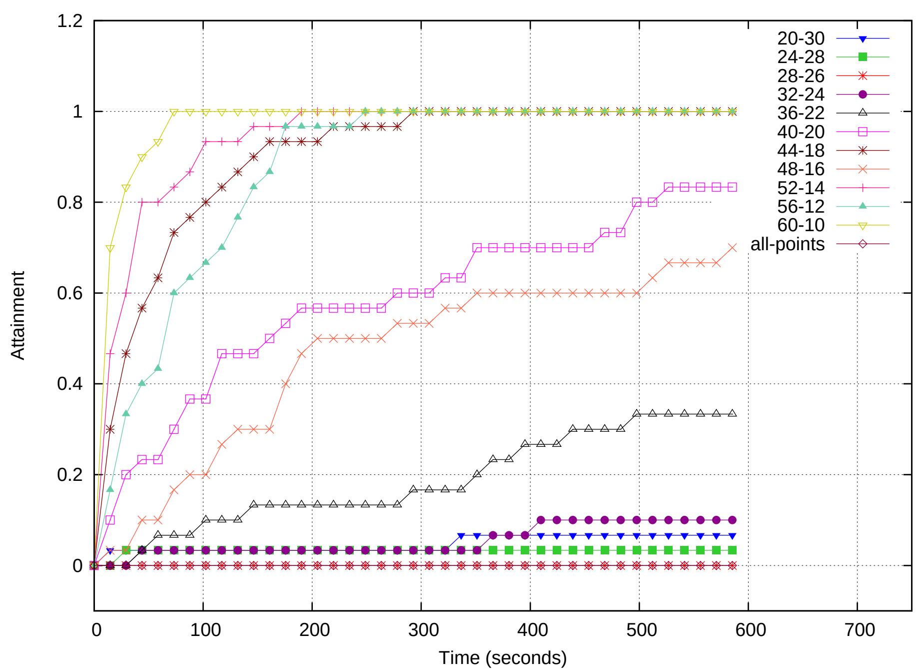
1.2
1
0.8
0.6
0.4
0.2
0
Attainment
0
100
200
300
400
500
600
700
Time (seconds)
20-3
40-2.9
60-1
all-points



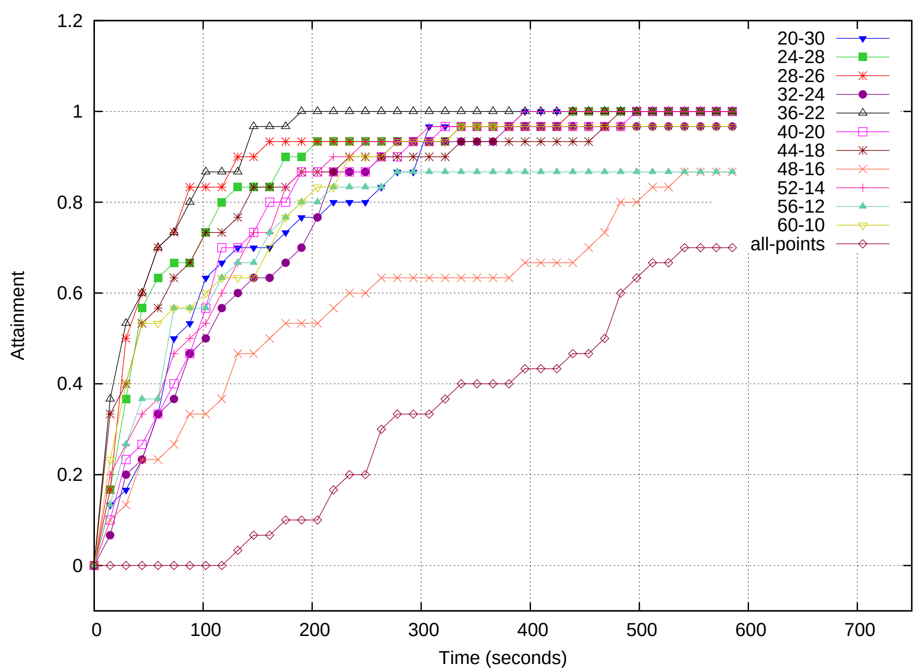
Attainment
Time (seconds)
1.2
1
0.8
0.6
0.4
0.2
0
0
150
300
450
600
750
20-30
24-26.2
28-22.4
32-18.6
36-14.8
40-11
44-10.8
48-10.6
52-10.4
56-10.2
60-10
all-points



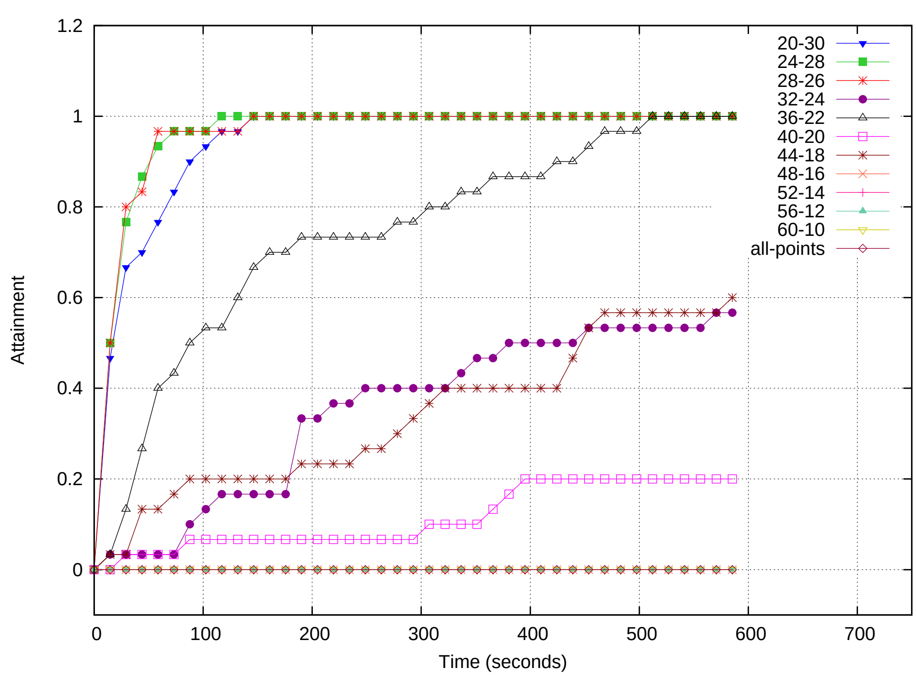
1.2
1
0.8
0.6
0.4
0.2
0
Attainment
0
150
300
450
600
750
Time (seconds)
20-30
24-28
28-26
32-24
36-22
40-20
44-18
48-16
52-14
56-13.8
60-10
all-points



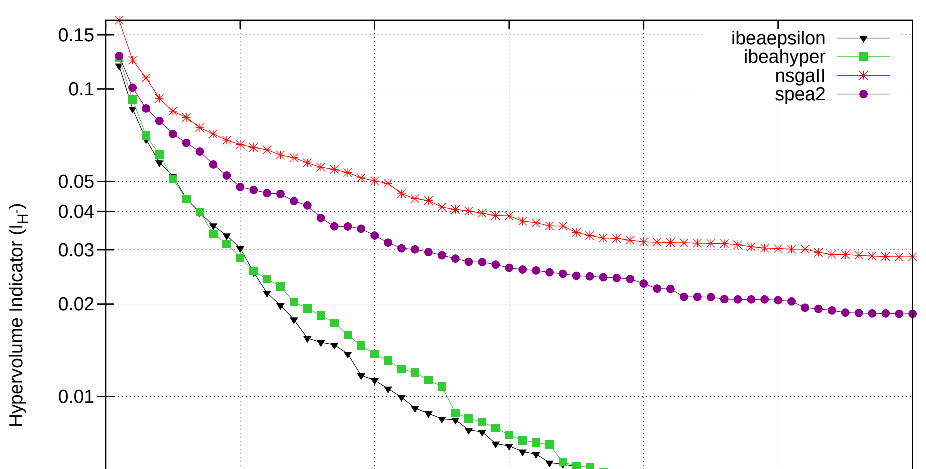
1.2
1
0.8
0.6
0.4
0.2
0
Attainment
0
100
200
300
400
500
600
700
Time (seconds)
20-30
24-28
28-26
32-24
36-22
40-20
44-18
48-16
52-14
56-12
60-10
all-points

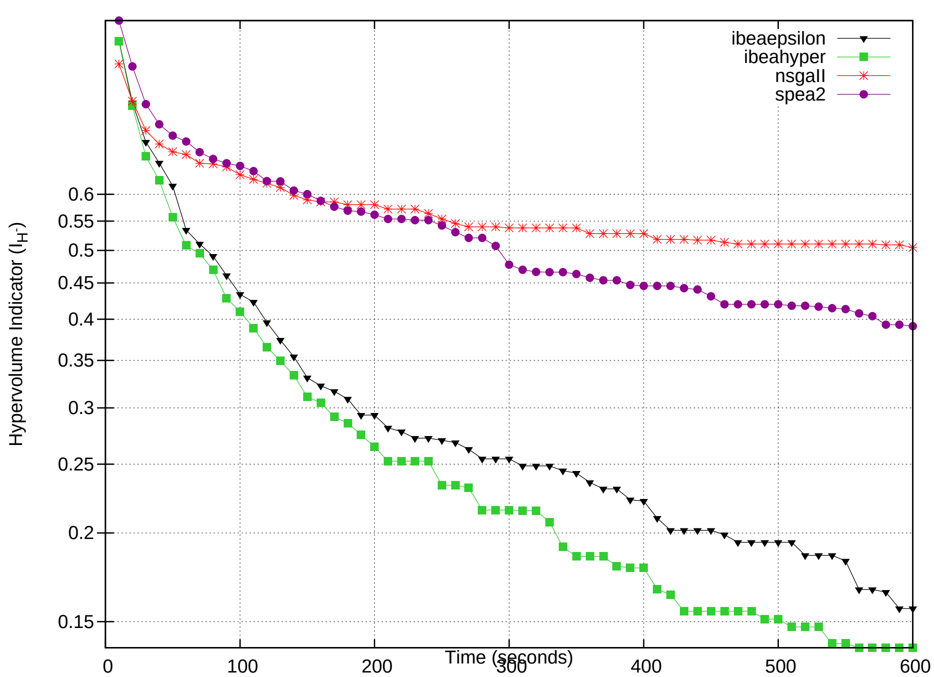


1.2
1
0.8
0.6
0.4
0.2
0
0
100
200
300
400
500
600
700
Attainment
Time (seconds)
20-30
24-28
28-26
32-24
36-22
40-20
44-18
48-16
52-14
56-12
60-10
all-points

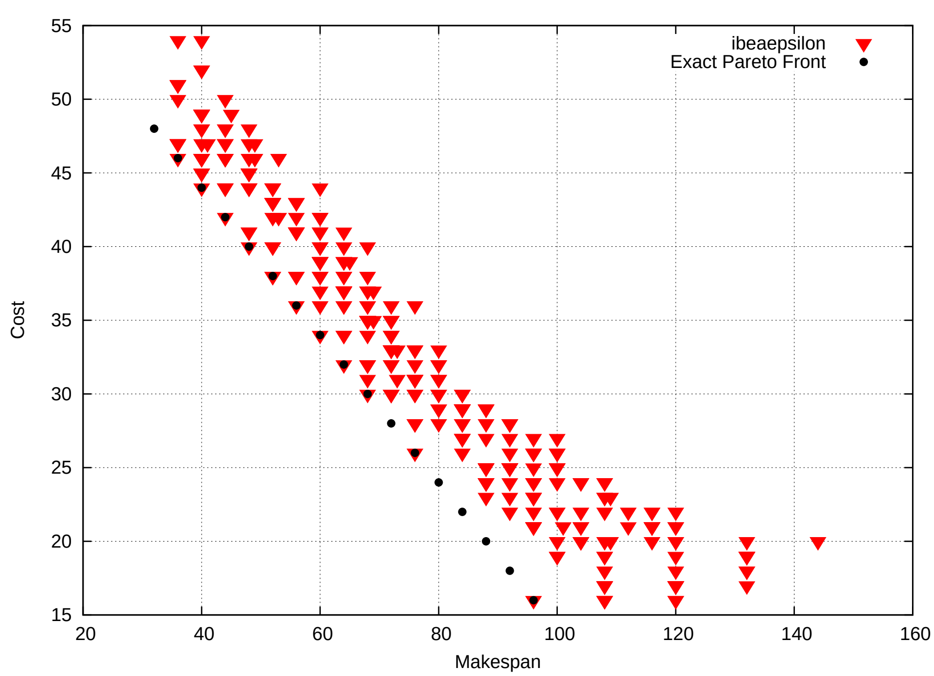


1.2
1
0.8
0.6
0.4
0.2
0
Attainment
0
100
200
300
400
500
600
700
Time (seconds)
20-30
24-28
28-26
32-24
36-22
40-20
44-18
48-16
52-14
56-12
60-10
all-points

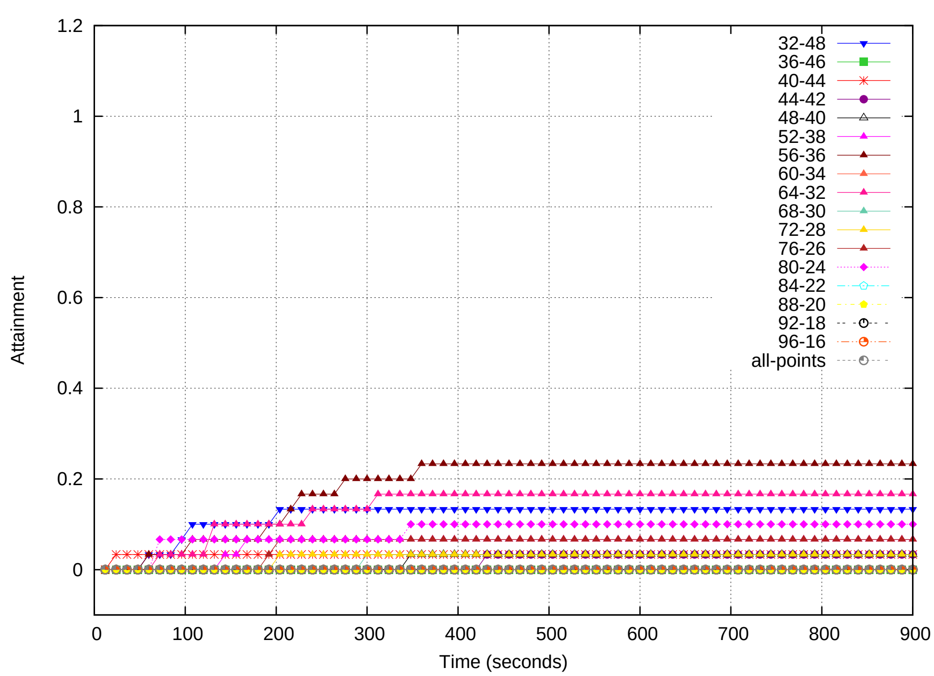

ibeaepsilon
ibeahyper
nsgaII
spea2
Hypervolume Indicator (I_{H^-})
0.15
0.1
0.05
0.04
0.03
0.02
0.01
0
100
200
300
400
500
600
Time (seconds)

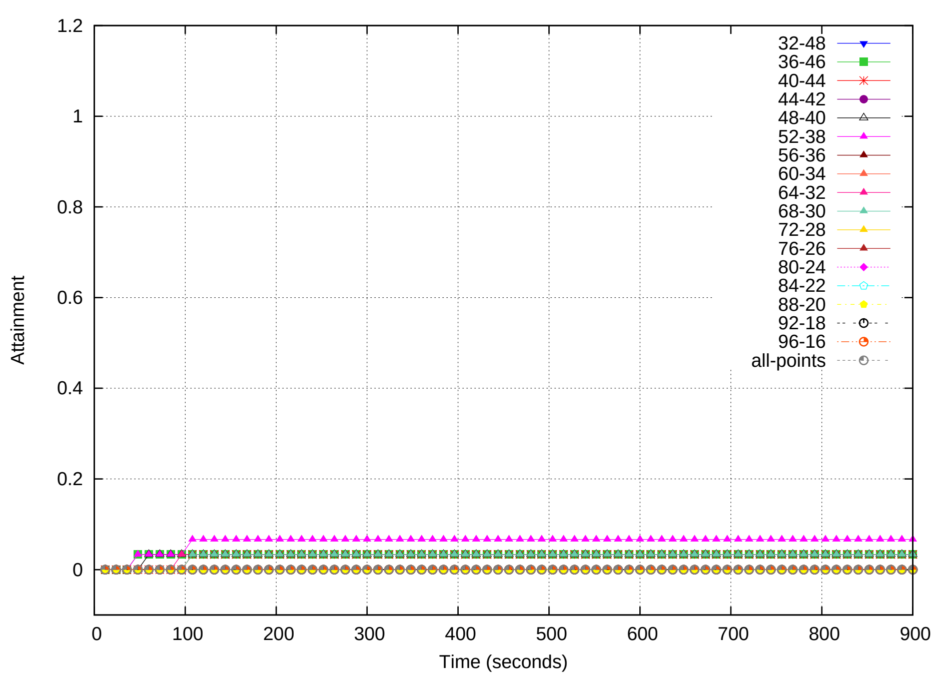


ibeaepsilon
ibeahyper
nsgaII
spea2
Hypervolume Indicator (I_{H^-})
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0
100
200
300
400
500
600
Time (seconds)

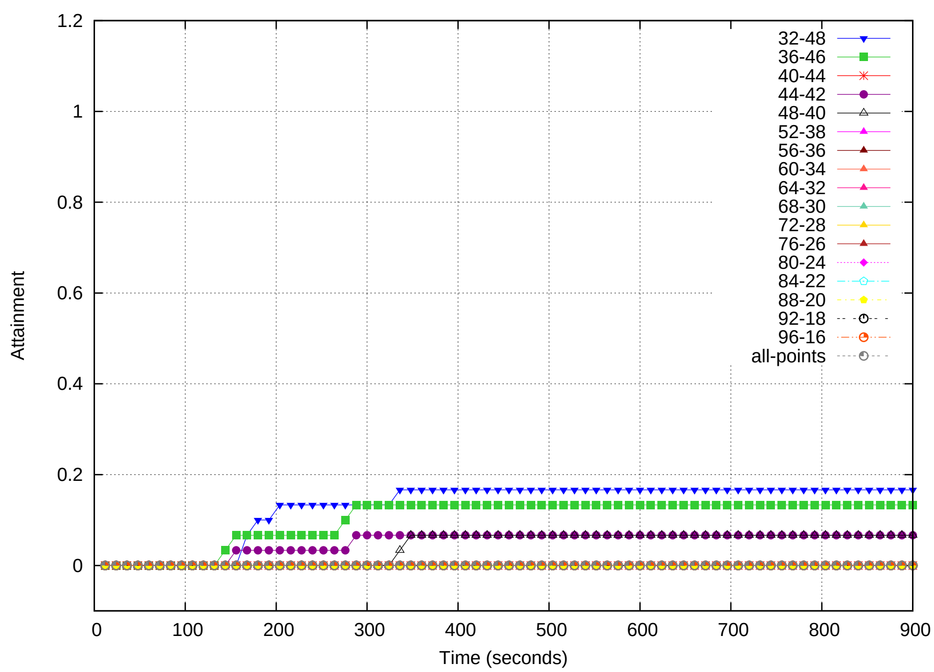


ibeaepsilon
Exact Pareto Front
Cost
Makespan
55
50
45
40
35
30
25
20
15
20
40
60
80
100
120
140
160

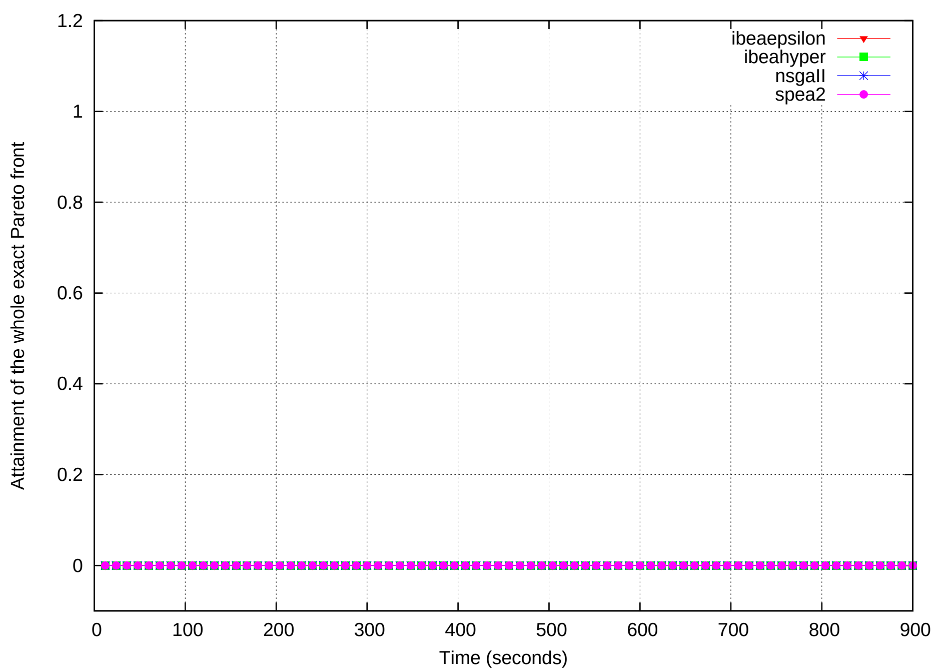

1.2
1
0.8
0.6
0.4
0.2
0
Attainment
0
100
200
300
400
500
600
700
800
900
Time (seconds)
32-48
36-46
40-44
44-42
48-40
52-38
56-36
60-34
64-32
68-30
72-28
76-26
80-24
84-22
88-20
92-18
96-16
all-points

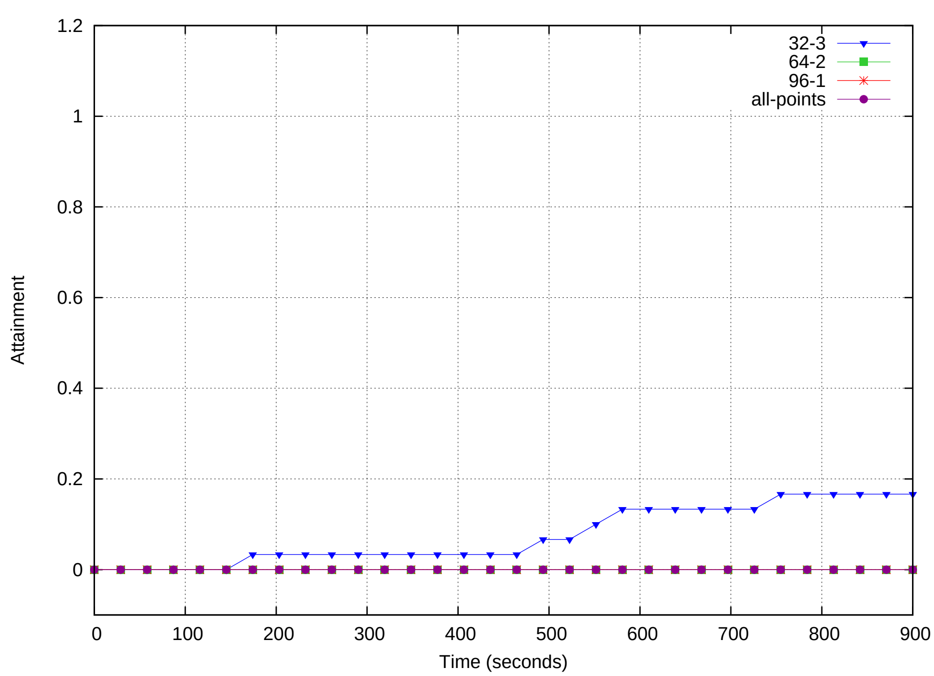


1.2
1
0.8
0.6
0.4
0.2
0
Attainment
0
100
200
300
400
500
600
700
800
900
Time (seconds)
32-48
36-46
40-44
44-42
48-40
52-38
56-36
60-34
64-32
68-30
72-28
76-26
80-24
84-22
88-20
92-18
96-16
all-points

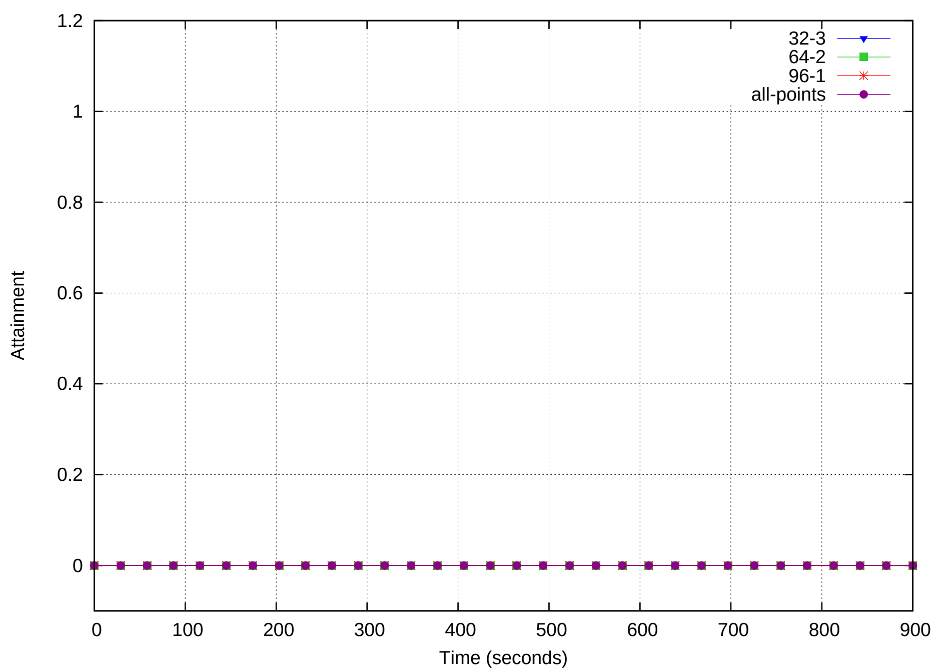


Attainment
Time (seconds)
1.2
1
0.8
0.6
0.4
0.2
0
0
100
200
300
400
500
600
700
800
900
32-48
36-46
40-44
44-42
48-40
52-38
56-36
60-34
64-32
68-30
72-28
76-26
80-24
84-22
88-20
92-18
96-16
all-points

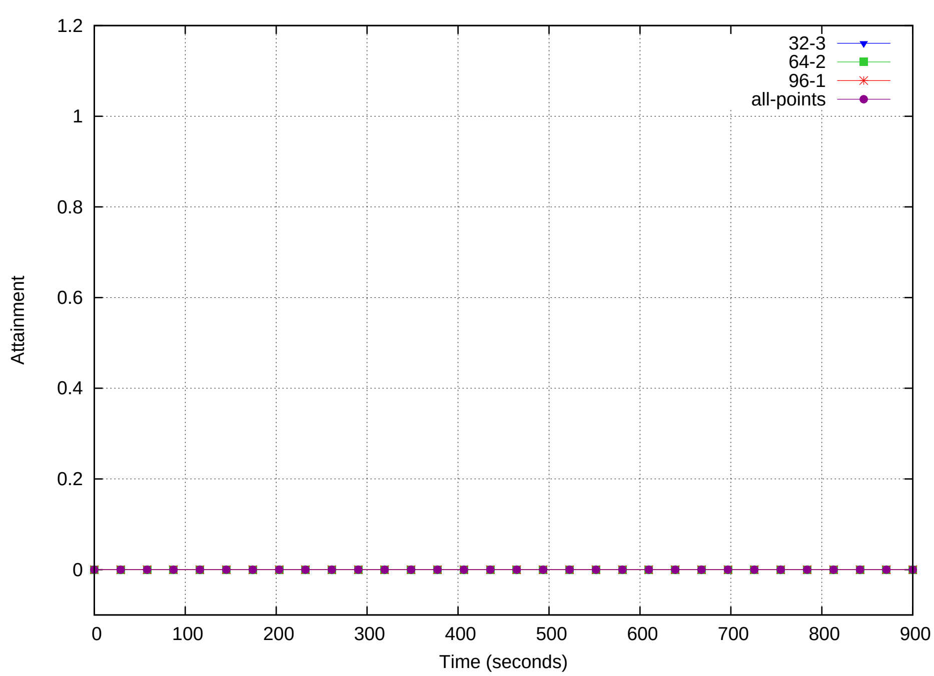

ibeaepsilon
ibeahyper
nsgaII
spea2
Attainment of the whole exact Pareto front
Time (seconds)
1.2
1
0.8
0.6
0.4
0.2
0
0
100
200
300
400
500
600
700
800
900



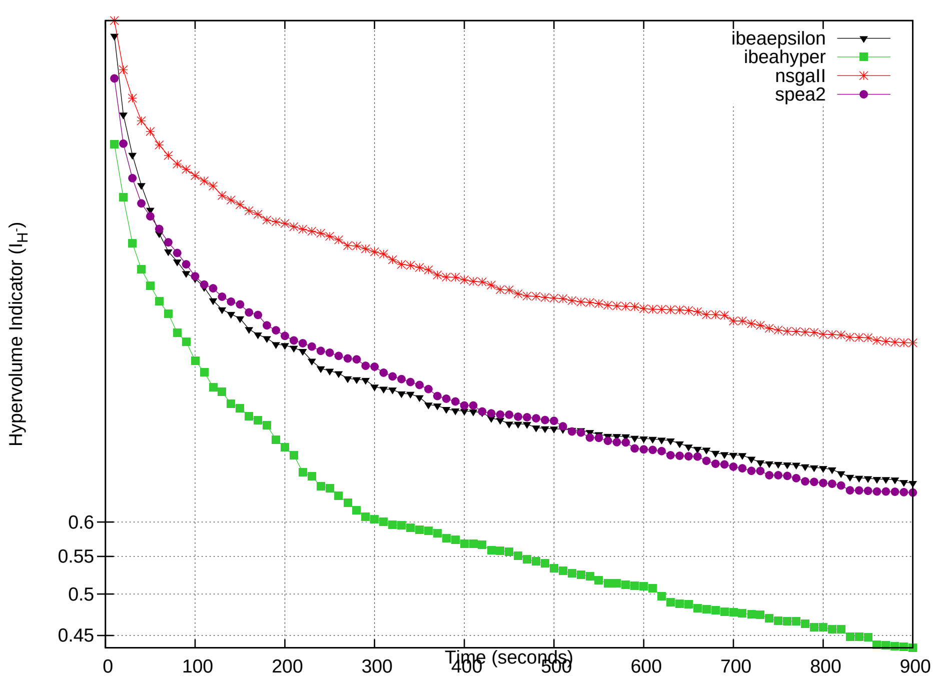
1.2
1
0.8
0.6
0.4
0.2
0
Attainment
0
100
200
300
400
500
600
700
800
900
Time (seconds)
32-3
64-2
96-1
all-points

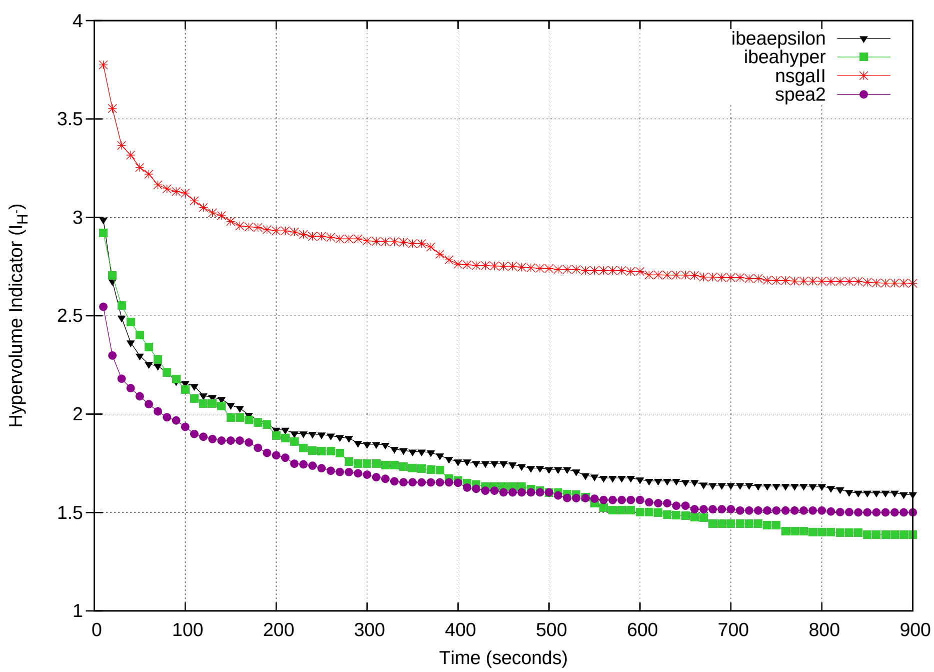


1.2
1
0.8
0.6
0.4
0.2
0
Attainment
0
100
200
300
400
500
600
700
800
900
Time (seconds)
32-3
64-2
96-1
all-points

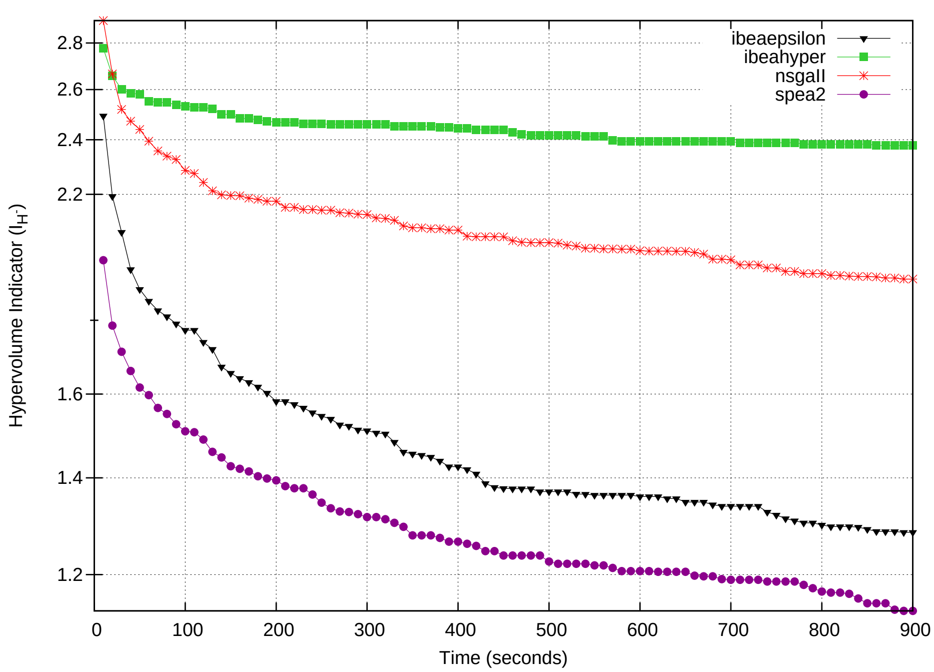


1.2
1
0.8
0.6
0.4
0.2
0
Attainment
0
100
200
300
400
500
600
700
800
900
Time (seconds)
32-3
64-2
96-1
all-points

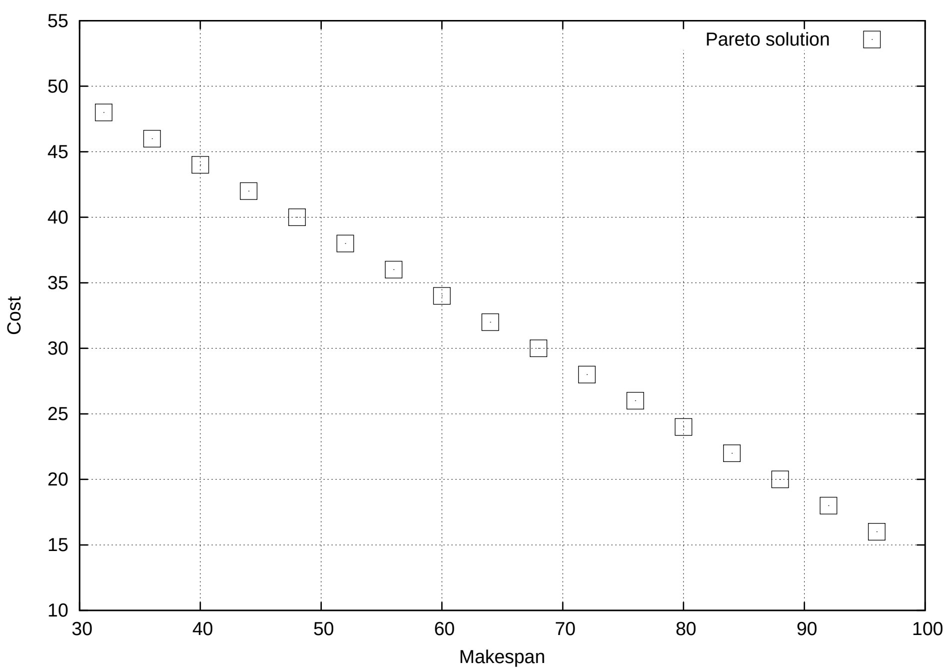

Hypervolume Indicator (I_H^-)
Time (seconds)
ibeaepsilon
ibeahyper
nsgaII
spea2
0.6
0.55
0.5
0.45
0
100
200
300
400
500
600
700
800
900



Hypervolume Indicator (I_{H^-})
Time (seconds)
ibeaepsilon
ibeahyper
nsgaII
spea2
4
3.5
3
2.5
2
1.5
1
0
100
200
300
400
500
600
700
800
900



ibeaepsilon
ibeahyper
nsgaII
spea2
2.8
2.6
2.4
2.2
1.6
1.4
1.2
Hypervolume Indicator (I_{H^-})
0
100
200
300
400
500
600
700
800
900
Time (seconds)


Pareto solution
Cost
Makespan
55
50
45
40
35
30
25
20
15
10
30
40
50
60
70
80
90
100